MATH 104 : Calculus I
Old Exam Problems

Name ____

Previous chapters

1. Compute the volume of the solid obtained by revolving the area enclosed by $y = x \ln x$, x-axis, $x = 1$ and $x = 3$ about the x-axis.

2. Compute the following integral

$$\int \sin^3 x \cos^5 x dx$$

3. Compute the following limits :

(a) $\lim_{x\to\infty} (x - \sqrt{x^2 + x})$ (b) $\lim_{x\to 0^+} \frac{e^{x^2} - 1}{x}$

4. Let $f(x) = \sqrt{x} \ln x$. Find the interval on which f is concave up.

5. Compute the following integrals using integration by parts.
(Hint : You might need to do a u-substitution first for some of them.)

(a) $\int e^{-2x} \sin(3x) dx$ (b) $\int x^3 \cos(x^2) dx$ (c) $\int_0^1 x^4 e^x dx$

6. Compute the following integrals using partial fractions :

(a) $\displaystyle\int_2^3 \frac{dx}{(x-1)^2(x^2+1)}$ (b) $\displaystyle\int \frac{-x^2+x-10}{(x-10)(x^2+9)}dx$ (c) $\displaystyle\int \frac{3x^2+8x+3}{x^3+2x^2+x}dx$

7. Compute the following integrals using a trigonometric substitution :

(a) $\displaystyle\int \frac{x^2}{\sqrt{16-x^2}}dx$ (b) $\displaystyle\int \frac{\sqrt{y^2-25}}{y^3}dy, \quad y>5$

8. Decide whether the following integrals are convergent or not :

(a) $\displaystyle\int_0^3 \frac{1}{x-2}dx$ (b) $\displaystyle\int_0^\infty \frac{\ln x}{x^2}dx$ (c) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{dx}{x\sin x}$ (d) $\displaystyle\int_1^\infty \frac{\sin^2 x}{x^2+1}dx$

Ch. 11

1. Find the equation for the line tangent to the curve defined by the parametrization

$$x = 1+\frac{1}{t}, \qquad y = t^3+3 \text{ (for } t>0)$$

at the point $(x,y)=(2,4)$.

(a) $y=-3x+10$ (b) $y=-3x+14$ (c) $y=3x-2$
(d) $y=3x-8$ (e) $y=-\frac{x}{3}+\frac{14}{3}$ (f) $y=-\frac{x}{3}+\frac{10}{3}$

2. Find an equation of the tangent to the curve

$$x = 2(t - \sin t), \qquad y = 2(1 - \cos t)$$

at the point

$$(x, y) = \left(\frac{2\pi}{3} - \sqrt{3}, 1\right).$$

(a) $y = \frac{\sqrt{3}}{4}x + \frac{7}{4} - \frac{\pi}{6}\sqrt{3}$ (b) $y = \frac{\sqrt{3}}{3}x + 2 - \frac{2\pi}{9}\sqrt{3}$

(c) $y = \frac{\sqrt{3}}{2}x + \frac{5}{2} - \frac{\pi}{3}\sqrt{3}$ (d) $y = \sqrt{3}x + 4 - \frac{2\pi}{3}\sqrt{3}$

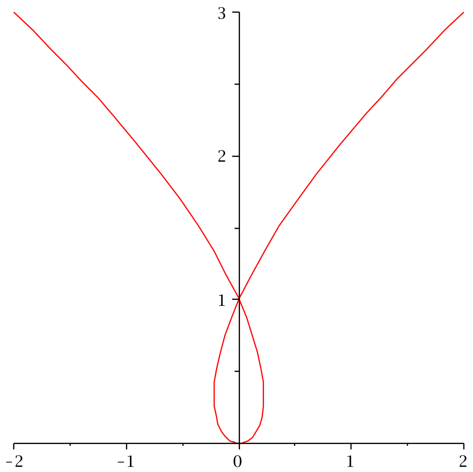
(e) $y = 2\sqrt{3}x + 7 - \frac{4\pi}{3}\sqrt{3}$ (f) $y = 3\sqrt{3}x + 10 - 2\pi\sqrt{3}$

3. A curve C is defined by the parametric equations

$$x = t - 3t^3, \qquad y = 3t^2.$$

The curve is given below. Notice that part of the curve contains a loop. Find the length of the loop.

(a) 0 (b) $\frac{8}{9}$ (c) $\frac{4\sqrt{3}}{3}$ (d) $\frac{16\sqrt{3}}{5}$ (e) $\frac{8\sqrt{3}}{15}$ (f) $\frac{16}{45}$



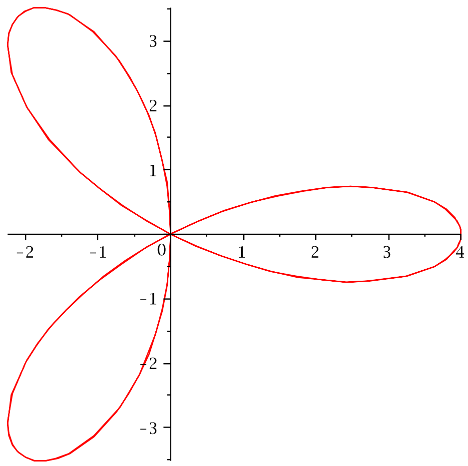
4. Find the length of the curve with parametric equations

$$x = \sin t + \cos t, \qquad y = \sin t - \cos t$$

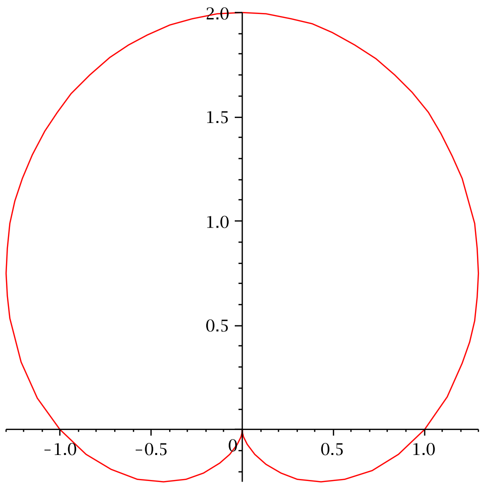
for $0 \le t \le 2\pi$.

5. Find the area inside *one* leaf (i.e., one loop) of the graph of $r = 4\cos 3\theta$.

(a) $\frac{2\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{4\pi}{3}$ (e) $\frac{7\pi}{4}$ (f) 2π



6. Find the length of the cardioid $r = 1 + \sin\theta$.



Ch. 12

1. Find the limit of the sequence $\left\{\left(\frac{1+n}{2+n}\right)^n\right\}$.

(a) $\frac{1}{2}$ (b) $\frac{1}{e}$ (c) 0 (d) 1 (e) e (f) sequence diverges

2. Find the limit of the sequence

$$\left\{\frac{1}{2}\ln(n^2+1) - \ln(2n+1)\right\}.$$

(a) -2 (b) $-\ln 2$ (c) 0 (d) $\ln 2$ (e) 2 (f) sequence diverges

3. Find the limit of the sequence

$$\lim_{n\to\infty} n \tan\frac{1}{n}$$

(a) 2 (b) 1 (c) -1 (d) $\frac{\pi}{2}$ (e) 0 (f) divergent

4. What is the limit of the sequence

$$\left\{\left(1+\frac{1}{n^2}\right)^n\right\}$$

(a) 1 (b) 1 (c) e (d) e^2 (f) does not exist

5. A sequence is defined by

$$a_1 = 2, \quad a_2 = 2^{1+1/2}, \quad a_3 = 2^{1+1/2+1/4}, \quad \ldots \quad a_n = 2^{1+1/2+1/2^2+\cdots+1/2^{n-1}}$$

Determine $\lim_{n\to\infty} a_n$.

(a) 0 (b) 1 (c) $\sqrt{2}$ (d) 2 (e) 4 (f) ∞

6. Determine if the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{3^n + 4^n}{7^n}$$

(a) 1 (b) $\frac{1}{7}$ (c) $\frac{5}{12}$ (d) $\frac{12}{7}$ (e) $\frac{25}{12}$ (f) diverges

7. How many of the following series converge?

$$\sum_{n=1}^{\infty} (\sqrt{2})^n \qquad \sum_{n=1}^{\infty} \frac{1}{n} \qquad \sum_{n=1}^{\infty} \left(\frac{e}{n}\right)^n \qquad \sum_{n=1}^{\infty} \frac{2^n - 1}{3^n} \qquad \sum_{n=1}^{\infty} \frac{n^2}{n^2+1}$$

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}+1}{n+2} \qquad \sum_{n=1}^{\infty} \frac{2^n + n^2}{3^n + n^3} \qquad \sum_{n=1}^{\infty} \frac{n^2+3}{(n+4)^2} \qquad \sum_{n=2}^{\infty} \frac{\ln n}{n^2} \qquad \sum_{n=1}^{\infty} \frac{n^2}{\ln n}$$

8. Does the series

$$\sum_{n=1}^{\infty} \frac{3 + \cos n}{2^n}$$

converge or diverge? Justify your answer.

9. Which statement is true of the following series?

$$\text{(I)} \quad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+1} = \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \cdots$$

$$\text{(II)} \quad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n!)^2}{(2n)!} = \frac{1}{2} - \frac{1}{6} + \frac{1}{20} - \cdots$$

$$\text{(III)} \quad \sum_{n=1}^{\infty} \frac{(-1)^n n^n}{n!} = 1 - 2 + \frac{9}{2} - \cdots$$

(a) (I) converges conditionally, (II) converges absolutely, (III) diverges.
(b) (I) converges absolutely, (II)converges conditionally, (III) diverges.
(c) (I) and (II) converge absolutely, (III) converges conditionally.
(d) (I) and (II) converge absolutely, (III) diverges.
(e) (I) and (III) diverge, (II) converges conditionally.
(f) (I) and (III) converge conditionally, (II) converges absolutely.

10. Determine whether the series is convergent or divergent. If the series is convergent, find its sum.

$$\sum_{n=0}^{\infty} e^{-3n}$$

(a) $\dfrac{1}{e-1}$ (b) $\dfrac{e}{e-1}$ (c) $\dfrac{1}{1-e^3}$ (d) $\dfrac{1}{e^3-1}$ (e) $\dfrac{e^3}{e^3-1}$ (f) divergent

11. Consider the following two series :

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n} \quad \text{and} \quad \sum_{n=2}^{\infty} \frac{(-1)^n}{ne^n}$$

Which of the following is true?

(a) both series are absolutely convergent.
(b) both series are conditionally convergent.
(c) both series are divergent.
(d) one series is absolutely convergent, and one series is conditionally convergent.
(e) one series is absolutely convergent, and one series is divergent.
(f) one series is conditionally convergent, and one series is divergent.

12. Consider the following three series :

$$\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}} \quad \text{and} \quad \sum_{n=1}^{\infty} \frac{1}{n^2+\sqrt{n}} \quad \text{and} \quad \sum_{n=1}^{\infty} \frac{1}{n^2+n}$$

Which of the following is true?

(a) only the first series converges.
(b) only the second series converges.
(c) only the third series converges.
(d) both the first and second series converge.
(e) both the first and third series converge.
(f) both the second and third series converge.

13. Determine which of the following series converge and which diverge :

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{(n+1)!n!2^n}$ (ii) $\displaystyle\sum_{n=1}^{\infty} ne^{-n^2}$ (iii) $\displaystyle\sum_{n=2}^{\infty} \frac{\ln(n)}{\ln(n^2)}$

14. Find the interval of convergence of

$$\sum_{n=2}^{\infty} \frac{1}{\ln n}(x-1)^n.$$

(a) $0 < x < 2$ (b) $-2 \le x < 2$ (c) $0 \le x \le 2$

(d) $0 \le x < 1$ (e) $0 \le x < 2$ (f) $1 - \frac{1}{e} \le x < 1 + \frac{1}{e}$

15. What is the largest open interval on which the series $\sum_{n=1}^{\infty} \frac{n(x-5)^n}{n^3+4^n}$ converges?

(a) $(-4, 4)$ (b) $(-4, 5)$ (c) $(4, 6)$ (d) $(1, 9)$ (e) $(4, 5)$ (f) $(-4, 14)$

16. Determine the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n3^n}.$$

(a) $[-1, 3)$ (b) $(-1, 3]$ (c) $[-1, 5)$ (d) $(-1, 5]$ (e) $[3, 5)$ (f) $(3, 5]$

17. Consider the power series

$$\sum_{n=1}^{\infty} \frac{(x-4)^n}{6^n n^{3/2}}$$

(a) What is the center (i.e., the point of expansion) for this series?

(b) What is the radius of convergence of this series?

(c) What is the interval of convergence for this series?

18. Find the first five terms of the Maclaurin series for

$$f(x) = \frac{x}{1-x^3}.$$

(a) $1 + x + x^2 + x^3 + x^4 + \cdots$
(b) $1 + x^3 + x^6 + x^9 + x^{12} + \cdots$
(c) $x + x^4 + x^7 + x^{10} + x^{13} + \cdots$
(d) $x^3 + x^4 + x^5 + x^6 + x^7 + \cdots$
(e) $x^3 + x^6 + x^9 + x^{12} + x^{15} + \cdots$
(f) $x + x^2 + x^3 + x^4 + x^5 + \cdots$

19. Which of the following is the beginning of the Maclaurin series for $\arctan(x^2)$?

(a) $x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4} + \cdots$
(b) $\frac{x^2}{3} - \frac{x^6}{6} + \frac{x^{10}}{9} - \frac{x^{14}}{12} + \cdots$
(c) $x^2 - 2x^4 + 3x^6 - 4x^8 + \cdots$
(d) $1 + 2x^2 + 3x^4 + 4x^6 + \cdots$
(e) $x^2 - \frac{x^6}{3} + \frac{x^{10}}{5} - \frac{x^{14}}{7} + \cdots$
(f) $x^2 + \frac{x^6}{2} + \frac{x^{10}}{3} + \frac{x^{14}}{4} + \cdots$

20. Which of the following is the beginning of the Maclaurin series for $\ln(1 + x^3)$?

(a) $x^3 - \frac{x^6}{2} + \frac{x^9}{3} - \frac{x^{12}}{4} + \cdots$
(b) $\frac{x^3}{3} - \frac{x^6}{6} + \frac{x^9}{9} - \frac{x^{12}}{12} + \cdots$
(c) $x^3 - 2x^6 + 3x^9 - 4x^{12} + \cdots$
(d) $1 + 2x^3 + 3x^6 + 4x^9 + \cdots$
(d) $\frac{x^3}{3} + \frac{x^6}{6} + \frac{x^9}{9} + \frac{x^{12}}{12} + \cdots$
(f) $x^3 + \frac{x^6}{2} + \frac{x^9}{3} + \frac{x^{12}}{4} + \cdots$

21. Find the Maclaurin series, and radius of convergence for

$$g(x) = \frac{x^3}{(1-x)^2}.$$

Justify your answer.

22. Find the first few terms of the Maclaurin series for

$$f(x) = \int \frac{\sin x}{x} dx$$

(a) $C + x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$
(b) $C + 1 - \frac{x^2}{(3)(3!)} + \frac{x^4}{(5)(5!)} - \frac{x^6}{(7)(7!)} + \cdots$
(c) $C + 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots$
(d) $C + 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$
(e) $C + x - \frac{x^3}{(3)(3!)} + \frac{x^5}{(5)(5!)} - \frac{x^7}{(7)(7!)} + \cdots$
(f) $C + x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

23. Let $F(x) = \int_0^x e^{-t^2} dt$. Which of the following is the beginning of the Maclaurin series for F?

(a) $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$
(b) $x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \cdots$
(c) $x - \frac{x^2}{2} + \frac{x^4}{6} - \frac{x^6}{24} + \cdots$
(d) $1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \cdots$
(d) $x - \frac{x^3}{3} + \frac{x^5}{15} - \frac{x^7}{105} + \cdots$
(f) $x + \frac{x^3}{3} + \frac{x^5}{15} + \frac{x^7}{105} + \cdots$

24. Which of the following is equal to $\ln x - \ln 3$ for x near 3 ?

(a) $\sum_{n=1}^{\infty} \frac{(x-3)^n}{3^n}$
(b) $\sum_{n=1}^{\infty} \frac{(-1)^n(x-3)^n}{n}$
(c) $\sum_{n=1}^{\infty} \frac{(-1)^n(x-3)^n}{n3^n}$
(d) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-3)^n}{3^n}$
(e) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-3)^n}{n3^n}$

25. In the Taylor series generated by $f(x) = x^{1/3}$ and centered at $a = 1$, the coefficient of $(x-1)^2$ is

(a) $-\frac{1}{9}$ (b) $\frac{1}{9}$ (c) $-\frac{2}{9}$ (d) $\frac{2}{9}$ (e) $-\frac{1}{3}$ (f) $\frac{1}{3}$

26. Let
$$\sum_{n=1}^{\infty} a_n x^n = (1-x)^{1/2} + (1+x)^{1/2}$$
be the Maclaurin expansion of $(1-x)^{1/2} + (1+x)^{1/2}$. Find
$$(a_0)^2 + (a_1)^2 + (a_2)^2.$$

(a) $\frac{65}{15}$ (b) $\frac{63}{16}$ (c) $\frac{7}{4}$ (d) $\frac{9}{4}$ (e) $\frac{81}{16}$ (f) $\frac{15}{4}$

27. Use the Taylor polynomial of order 5 of $\sin(x^2)$ to approximate
$$\int_0^1 \sin(x^2)dx$$

28. (a) Write the second-degree Taylor polynomial for $f(x) = \sqrt{x}$ centered at $a = 100$.

(b) Use the polynomial from (a) to estimate $\sqrt{101}$. Also, give an estimate of the error.

29. Evaluate $\int \frac{e^x}{x} dx$ as an infinite series.

30. Let
$$f(x) = e^{2x^3} + \cos(3x^2).$$
Find $f^{(9)}(0)$ (the ninth derivative of f evaluated at 0).
(Hint : Don't even think of differentiating the function nine times.)