Math 180, Fall 2014 Assignment 4

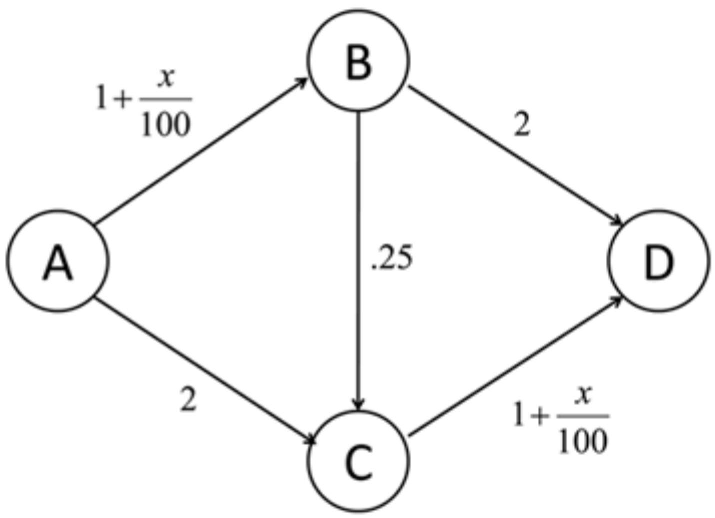
This assignment is due Tuesday, September 30.

Exercise 1. "Braess's paradox, credited to the German mathematician Dietrich Braess, states that adding extra capacity to a network when the moving entities selfishly choose their route, can in some cases reduce overall performance." (Wikipedi), It is a relatively recently discovered phenomenon which has actually been observed in traffic. It would make a good topic for a presentation. Please let me know if any team is interested. Please do the exercise on Braess' paradox on the following page.

Exercise 2. Although we are not yet finished with Game Theory, the following simple exercise brings us into the next unit, Financial Planning. If you are not already familiar with the workings of compound interest, you can easily find many sources besides our text.

Suppose that you have been fortunate enough to win the powerball lottery and are entitled to \$50 million "annuitized": you have a choice of an immediate payment of \$25 million or \$2 million a year for the next 25 years. Assume an interest rate of 5% per year. Which should you choose?

Here is an application of the concept of Nash equilibrium in determining the expected flow of traffic in a network. Consider the network of roads below in which cars must travel from A to D. (Notice that the roads are all one-way.) Assume that 100 cars must make the trip and that the travel times from A to C and from B to D are each 2 hours and that from B to C is ¼ hour, independent of the number of cars that travel these roads. However, the travel time from A to B depends on the number of cars that choose that road: if the number of cars is x then the time it takes is 1 +x/100 hours, and likewise for the link CD. (The "x" on the AB link is not necessarily the same as on the CD link. For each link it is just the number of cars traveling on that road.) What is the expected distribution of traffic in the network?



We may view this as a game in which the travelers are the players and each has a choice of 3 strategies, namely, the paths ABD, ACD, and ABCD. The "payoff" (actually penalty) for each strategy is the travel time of the chosen route. The payoffs for any given strategy depend on the choices of the other players, as is usual. The goal in this case is to minimize travel time. Equilibrium will occur when no single driver has any incentive to switch routes, since it can only add to his/her travel time. If, for example, 100 cars are traveling from A to D, then equilibrium will occur when 25 drivers travel via *ABD*, 50 via *ABCD*, and 25 via *ACD*. Every driver now has a total travel time of 3.75. This distribution is not socially optimal. If the 100 drivers agreed that 50 travel via *ABD* and the other 50 through *ACD*, then travel time for any single car would actually be only 3.5 hours. This is an example of Braess' paradox: Sometimes closing a road actually decreases travel time. In this picture, if the number of travelers is small (suppose, for example, there were only one) then the link BC is useful (for he could then choose ABCD and make the trip in 2.27 hours, far less than ABD or ACD).

1. Is the distribution where 50 drivers choose *ABD* and 50 chose *ACD* an equilibrium? 2. Show that the distribution above (25 drivers traveling via *ABD*, 50 via *ABCD*, and 25 via *ACD)* actually is an equilibrium. Is there any other? 3. Show that if the number of travelers is large enough then at equilibrium no one will use the link BC. 4. There is a range in which, because of the link BC, the equilibrium is not socially optimal and it would be better to close it. What is that range? (For example, with 100 travelers it is better to close BC.)