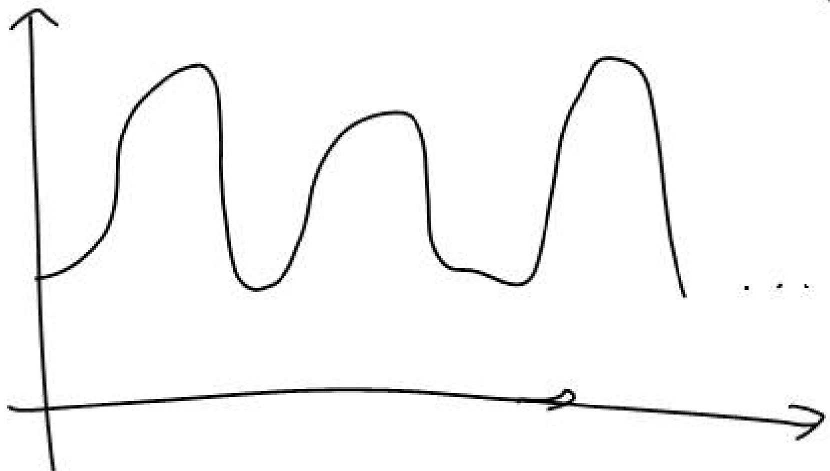
8.3

#1.



Kiwi price changes

Kind of Periodic (sin x)

Understand: kiwi price.

periodic functions

function as a sum of trigonometric fns.

Examples:

- Fourier transform (GPS, audio)
- Time series: (GDP estimate)
- Elementary particles wave decomposition

Goal. $\int \sin^n x \cos^m x\, dx$ or $\int \tan^n x \sec^m x\, dx$

How to integrate $\sin^n x \cos^m x$?

Case 1: n is odd. $n = 2k+1$

$$\sin^n x = \sin^{2k+1} x = \sin x \cdot \sin^{2k} x = \sin x \cdot (\sin^2 x)^k$$

$$\sin x \cdot (\sin^2 x)^k = \sin x (1-\cos^2 x)^k$$

$$\int \sin^n x \cos^m x\, dx = \int \sin x \cdot (1-\cos^2 x)^k \cos^m x\, dx$$

$u = \cos x$

$du = -\sin x\, dx$

$$= -\int (1-u^2)^k u^m\, du$$

$n \quad s \quad 3 \qquad n = 2 \quad + \cancel{4}$

$$\int \sin^3 x \cos^4 x\, dx = \int \sin x \cdot \sin^2 x \cdot \cos^4 x\, dx = \int \underline{\sin x} (1-\cos^2 x) \cos^4 x\, \underline{dx}$$

$\| \; u = \cos x$

$du = -\sin x\, dx$

$$-\frac{\cos^5 x}{5} + \frac{\cos^7 x}{7} + C, = -\frac{u^5}{5} + \frac{u^7}{7} + C = -\int u^4 - u^6\, du = \int (1-u^2)\, u^4\, (-du)$$

Case 2: m is odd (same story, separate cos x & write:

$$\int \cos x \cdot (\text{polynomial in } \sin x)\, dx$$

u-sub $u = \sin x$.

Case 3: m and n are even. $\int \sin^{10} x \cos^{6} x \, dx$ (① write everything in terms of cos x)

$\sin^{10} x = (\sin^2)^5 = (1-\cos^2)^5$

$= (1-\cos^2)^5$

$$\int \underbrace{\cos^2 2x}_{\frac{\cos 4x + 1}{2}}\, dx = \int \frac{\cos 4x + 1}{2}\, dx$$

$$\frac{\sin 4x}{8} \qquad \frac{x}{2}$$

$$\int \cos 2x \cdot \cos^2 2x\, dx = \int \cos 2x\,(1 - \sin^2 2x)\, dx$$

$$\int \frac{1-u^2}{2}\, du \qquad u = \sin 2x \quad du = 2\cos 2x$$

Identity: $\cos^2 x = \dfrac{\cos(2x) + 1}{2}$

Now: $\displaystyle\int \cos^{2k}(x)\, dx = \int \left(\frac{\cos 2x + 1}{2}\right)^k dx$ Foil out

Example: $\displaystyle\int \cos^6 x\, dx = \int (\cos^2)^3 dx = \int \left(\frac{\cos 2x + 1}{2}\right)^3 dx =$

$$= \frac{1}{8} \int \underbrace{\cos^3 2x}_{\int \cos 2x \cdot \cos^2 2x\, dx} + 3 \cdot \cos^2 2x + \underbrace{3\cos 2x}_{\frac{3}{2}\sin 2x} + \underbrace{1}_{x}\, dx = \frac{1}{8}\left(\frac{\sin 2x - \frac{\sin^3 2x}{3}}{2} + \frac{3}{8}\sin 4x + \frac{3x}{2} + \frac{3}{2}\sin 2x + x + C \right)$$

$$\int \tan^n x \sec^m x \, dx$$

Case 1: n is odd · you separate $\tan x$, identity $\tan^2 x = \sec^2 x - 1$

$$\tan^2 x = \sec^2 x - 1$$

$$\int \tan x \sec^k x \, dx = \int \tan x \sec x \cdot \sec^{k-1} x \, dx = \int u^{k-1} du$$

$u = \sec x$

$du = \tan x \sec x \, dx$

Case 2: m is even : you separate $\sec^2 x$, $\tan^2 x + 1 = \sec^2 x$

write integral:

$$\int \sec^2 x \, (\text{polynomial in } \tan x) \, dx$$

$u = \tan x$

C. 3: n is even, m is odd $(\tan^2 x = \sec^2 x - 1)$ eliminate all $\tan x$

$u = \sec x$
$v' = \sec^2 x$
$u' = \sec x \tan x$
$v = \tan x$

Case 3: Example.

$$\int \sec^3 x \, dx = \int \underbrace{\sec x}_{u} \cdot \underbrace{\sec^2 x}_{\substack{v' \\ \tan'}} \, dx \quad \text{IBP}$$

$$\sec x \tan x - \int \sec x \tan x \tan x = uv - \int u'v$$

$$\sec x \tan x - \int \sec x \underbrace{\tan^2 x}_{\sec^2 x - 1} \, dx = \sec x \tan x - \underbrace{\cancel{\int \sec^3 x}}_{\sec^3 x - \sec x} + \int \sec x$$

$$2 \int \sec^3 x \, dx = \sec x \tan x + \int \sec x = \sec x \tan x + \ln|\sec x + \tan x| + C$$

$$\int \sec^3 x \, dx = \frac{1}{2}\left(\sec x \tan x + \ln|\sec x + \tan x| \right) + C$$

8.4.

$1+x^2$: $x=\tan u$: $1+x^2 = 1+\tan^2 u = \sec^2 u$

$1-x^2$: $x=\sin u$ $1-x^2 = 1-\sin^2 u = \cos^2 u$

x^2-1 : $x=\sec u$ $\cancel{\text{so}}$ $x^2-1 = \sec^2 u - 1 = \tan^2 u$

#8. Arc length:

$$\int_a^{\cancel{b}} \sqrt{f'(x)^2+1}\,dx = \int_0^4 \sqrt{4x^2+1}\,dx = \int \sqrt{\tan^2 u+1}\cdot\frac{\sec^2 u}{2}\,du$$

$2x=\tan u$

$2dx = \sec^2 u\,du$

$f(x)=x^2$ $f'(x)=2x$ $f'(x)^2=4x^2$

$$= \int \frac{\sec^3 u}{2}\,du$$

$$\frac{1}{4}\left(\sec u\tan u + \ln|\sec u+\tan u|\right)\Big]_{u=\tan^{-1}(0)}^{u=\tan^{-1}(8)}$$