WORKSHEET 13

1. Use the comparison theorem to deduce that the integrals are convergent or divergent.

   - $\int_0^\infty \sin^2(t)e^{-t}dt$
   - $\int_0^\infty e^{t+\sin^2 t}dt$
   - $\int_0^\infty \frac{1}{x+\sqrt{x}+1}dx$
   - $\int_0^\infty \frac{x^2}{x^2+e^x+2}dx$

2. Check whether the following functions are probability density functions.

   - $f(x) = -x^2$
   - $f(x) = \begin{cases} 1/2 & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$
   - $f(x) = \begin{cases} 1/2 & \text{if } 0 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$
   - $f(x) = \begin{cases} 2x & \text{if } -1 \leq x \leq \sqrt{2} \\ 0 & \text{otherwise} \end{cases}$
   - $f(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$

3. Find the value c so that $f(x) = c\frac{1}{1+x^2}$ is a probability density function.

4. Exponential distribution

   - Show that $f(x) = \begin{cases} 3e^{-3x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$ is a probability density function.
   - What is its mean?
   - What is its median?
   - What is its standard deviation?

5. Uniform distribution

- Show that $f(x) = \begin{cases} \frac{1}{5} & \text{if } -2 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$ is a probability density function.
- What is its mean?
- What is its median?
- What is its standard deviation?

6. Normal distribution: Use wolfram alpha or other computer program to verify the following statements. (open up http://www.wolframalpha.com and for instance type in the expression int_{- infinity}^{infinity} e^{-(x-2)^2/18}dx)

   - Show that $f(x) = \frac{1}{3\sqrt{2\pi}} e^{\frac{-(x-2)^2}{18}}$ is a probability density function.
   - Its mean is 2.
   - Its standard deviation is 3.

7. Marci is waiting for a bus in Hungary. The average waiting time is 6 minutes. Assume that the random variable corresponding to the waiting time is exponential (meaning that the probability density function is

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

   for some number λ). What is the probability that Marci will wait at least 10 minutes for the bus? Hint: first calculate λ.

8. The biggest discovered atom is the Ununoctium. It has no stable isotopes, the first and only isotope synthesized is ^{294}-Ununoctium which has average life span 890 microsecond. Assume that the random variable corresponding to the life of ^{294}-Ununoctium is exponential. In other words, the mean of the random variable is 890 microsecond. What is the probability that ^{294}-Ununoctium will be stable for at least 1 second?