WORKSHEET 7

"Physics and engineering and economics frequently involve products. Work is force times distance. Power is voltage times current. Income is price times quantity." It is very important to learn how to deal with these kinds of integrals.

1. Check whether the following functions are odd or not.

   - $x^2 \sin(x)$
   - x^4
   - $\sin^3(x)$.

   Compute their integral over $[-1, 1]$.

2. Compute the x-coordinate of the centroid of the region in the first quadrant bounded by $y = \cos x$ and $y = \sin x$ and $x = 0$.

3. Use tabular integration to calculate:

   - $\int x^2 \sin(x) dx$
   - $\int x^3 e^{2x} dx$
   - $\int 3x \sec^2(x) dx$.

4. Use $v' = 1$ in order to compute the following integrals:

   - $\int \ln(x) dx$
   - $\int \ln^2(x) dx$
   - $\int \sin^{-1}(x) dx$ Hint: $\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$.

5. Use integration by parts twice and solve the equation for $\int e^x \sin(2x) dx$. Hint: you can use $u = e^x$.

6. Use the shell method to calculate the volume of the solid generated by revolving the region in the first quadrant bounded by $y = e^x$ and $x = \ln 2$ about the y-axis.

7. Hard one: Use integration by parts to calculate $\int \frac{\ln(x)}{x} dx$. Let $u = \frac{1}{x}$ and $v' = \ln(x)$. What do you get? $0 = -1$??? What???