Name: ________________________________

Midterm Exam #2 for Math 370

Dec 8, 2015

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

- You have an hour and twenty minutes for this exam.
- Please show **ALL** your work on this exam paper. Partial credit will be awarded where appropriate.
- You may quote without proof the theorems/propositions/lemmas given during lecture as long as you state each result clearly. Points may be deducted for incorrect or missing hypotheses.
- **NO** calculators, books, notes, or computers are permitted.
- **NO** form of cheating will be tolerated. You are expected to uphold the Code of Academic Integrity.

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this midterm examination.

Name (printed): ________________________________

Signature: ________________________________

Date: ________________________________

1.

Consider the permutations (123) and (23)(45) in S_6

(a) (4 points) Show that the conjugate of (123) by (23)(45) is (132).

(b) (2 points) Show that the subgroup generated by (123) and (23)(45) has 6 elements.

(c) (4 points) Show that it is isomorphic to S_3.

(You can use any convention of multiplication of permutations.)

2. Consider the matrix

$$\begin{pmatrix} 3/5 & 0 & 4/5 \\ 3/5 & 1 & -6/5 \\ -1/5 & 0 & 7/5 \end{pmatrix}$$

a) (5 points) Compute its characteristic polynomial.

b) (5 points) Compute its minimal polynomial.

Extra space for work:

3.

a) (6 points) Let $\phi : V \to V$ be a linear transformation, so that $\phi^2 = \phi$. Prove that $V = ker\phi \oplus im\phi$.

b) (4 points) Let V be a finite dimensional vector space and U be a subspace. Find a $\phi : V \to V$ linear transformation, so that $\phi^2 = \phi$ and $\ker \phi = U$.

Extra space for work:

4.

a) (5 points) Let H be a subgroup of G. Show that two cosets of H are either the same or distinct.

b) (5 points) State and prove Lagrange's Theorem.

Extra space for work:

5. Consider the regular n-gon in the plane, and enumerate the vertices by $1, 2, ..., n$. In this way we get an embedding of D_n into S_n (every rotation or reflection can be thought of as a permutation on the vertices). Let us denote the set $D_n \cap A_n$ inside S_n by AD_n.

a) (2 points) Show that AD_n is a subgroup of D_n.

b) (4 points) Show that $|D_n : AD_n| \leq 2$.

b) (2 points) Compute the size of AD_4.

c) (2 points) Compute the size of AD_5.

Extra space for work: