HOMEWORK 1

1. Construct the truth tables for the following statements. Determine whether each is a tautology, a contradiction or neither:

   - $\neg(P \wedge Q) \rightarrow (\neg P \wedge \neg Q)$
   - $(P \wedge Q) \vee (\neg P \wedge R)$
   - $((P \vee Q) \wedge R) \leftrightarrow (P \vee (Q \wedge R))$
   - $\neg(P \leftrightarrow Q) \leftrightarrow (\neg P \leftrightarrow Q)$
   - $(P \wedge Q) \wedge (\neg P \wedge \neg Q)$

2. In class I mentioned that the "or" we use, $\vee$ is different from what we may use in daily language. The other "or" (the so called exclusive or) means that at most one of the statements can be true. For instance "I will study tonight or I will go to a party." means that only one of the two possibilities can happen.

   - Construct the truth table for exclusive or.
   - Write down a statement using only the 5 standard connectives which is equivalent to the exclusive or.

3. For each of the following, replace the symbol $*$ with a connective so that the resulting statement is a tautology. Justify your answer.

   - $(P \rightarrow (Q * R)) \leftrightarrow ((P \rightarrow Q) \wedge (P \rightarrow R))$
   - $((P * Q) \rightarrow R) \leftrightarrow ((P \rightarrow R) \wedge (Q \rightarrow R))$
   - $((P * Q) \rightarrow R) \leftrightarrow ((P \rightarrow (Q \rightarrow R))$

4. Negate the following statements.

   - Pigs are not blue or dogs are not green.
   - Pigs are not blue and dogs are not green.
   - If pigs are not blue, then dogs are not green.
   - Pigs are not blue if and only if dogs are not green.
   - If pigs are not blue or dogs are not green, then pigs are not green and dogs are not blue.
   - Pigs are not dogs.

5. Show that the following arguments are valid without using truth tables.

   - Hypotheses: $Q \to R$, $R \vee S \to P$, $Q \vee S$. Conclusion: P.
   - Hypotheses: $P \to Q$, $P \to \neg R$, $Q \leftrightarrow R$. Conclusion: $\neg P$.

6. Show that the following argument is not valid: Hypotheses: $P \vee Q$, $(Q \vee R) \leftrightarrow \neg P$. Conclusion: $R \vee \neg Q$.

7. Starting with any positive number, we generate a sequence of numbers by the following rules: If the current number is even, the next number is half the current number. If the current number is odd, the next number is 1 more than 3 times the current number.

   - Choose three starting numbers between 25 and 50 and for each of them generate the sequence until it stabilizes.
   - On the basis of your result, formulate a conjecture about the sequences.

   (You do not need to prove the conjecture.)