HOMEWORK 2

1. Represent each of the following statements symbolically, starting with only the following atomic statements: $P(x, y)$ for "x is a parent of y," $W(x)$ for "x is female," and $x = y$ meaning "x and y are the same person." All of your variables should have the set of all people as their domain. You may introduce new propositional variables for statements that you have already written symbolically in a previous part of this question.

   (a) x is y's father.

   (b) x is y's grandmother.

   (c) x is y's sibling (meaning x and y have the same parents, but they are not the same person).

   (d) x is an only child (i.e. x has no siblings).

   (e) x is y's first cousin.

   (f) x has no uncles.

2. Translate the following into symbolic notation. Temporal statements are translating by introducing time t as an additional mathematical variable. As an example, one can define a function $L(x, y, t)$ to stand for "x loves y at time t."

   (a) Not everyone likes spinach, and no one likes asparagus.

   (b) All crows are black, but not all black things are crows.

   (c) It is possible to fool all of the people some of the time and some of the people all the time, but not all people all the time.

   (d) Everybody loves somebody sometimes.

   (e) It is not true in all cases that if one person likes another, the second person likes the first.

3. Determine whether each of the following statements is true or false if all variables have the set of real numbers as their domain. Justify your answer.

   (a) $\forall x\, \exists y\, (x^2 = y)$

   (b) $\forall y\, \exists x\, (x^2 = y)$

   (c) $\exists x\, \forall y\, (x + 5 = y)$

(d) $\forall x\, \forall y\, \exists z\, \forall u\, (x + z = y + u)$

(e) $\forall x\, \forall y\, \exists z\, (x^2 + y^2 = z^2)$

(f) $\exists x\, [\forall y\, (yx^2 = y) \wedge \neg \forall y\, (yx = y)]$

4. There is a first order language for plane (Euclidean) geometry. It consists of three sorts of objects: points, lines, and nonnegative real numbers (denoting magnitudes). Operator symbols $+$, $-$, $\times$, and $/$ are included for use on nonnegative real numbers only. Predicate symbols $=$ (referring to all three sorts), $<$, and $>$ (referring to nonnegative real numbers) are also included. In addition, the predicate symbol $On(A, L)$ means point A is on line L. The operator symbol $\overline{AB}$ denotes the line segment between points A and B, whereas the operator symbol $|\overline{AB}|$ denotes the length of that line segment. Let A, B, and C denote points. Let L and M denote lines. Translate the following into symbolic notation making use of the quantifier $\exists!$ (wherever appropriate) in addition to the universal and existential quantifiers:

   (a) Lines L and M are parallel, i.e. they have no point in common.

   (b) Any two *distinct* lines meet in *at most* one point.

   (c) Given any two distinct points, there is one and only one line passing through both of them.

   (d) Given any line and any point not on that line, there exists one and only one line through that point that is parallel to the given line. (This is one version of Euclid's Postulate/The Parallel Postulate)

   (e) C is the midpoint of the line segment $\overline{AB}$. (Do not forget to specify that C is on the line determined by A and B.)

5. Simplify each of the following statements by moving negation signs inward as much as possible:

   (a) $\neg \forall\, x, y\, \exists\, z\, (P \vee \neg \forall\, u\, Q)$

   (b) $\neg \forall\, x\, \neg \exists\, y\, \neg \forall\, z\, (P \wedge \neg Q)$