Name: ______________________________

Midterm Exam #1 for Math 371

Feb 11, 2016

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

- You have an hour and twenty minutes for this exam.
- Please show **ALL** your work on this exam paper. Partial credit will be awarded where appropriate.
- You may quote without proof the theorems/propositions/lemmas given during lecture as long as you state each result clearly. Points may be deducted for incorrect or missing hypotheses.
- **NO** calculators, books, notes, or computers are permitted.
- **NO** form of cheating will be tolerated. You are expected to uphold the Code of Academic Integrity.

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this midterm examination.

Name (printed): ______________________________

Signature: ______________________________

Date: ______________________________

1.

(a) (4 points) Show that the set of 2-by-2 real matrices whose 21 entry is 0 is a ring with respect to the usual addition and multiplication of matrices.

b) (3 points) Give an example of a unit in this ring (which is not the identity matrix).

(c) (3 points) Give an example of a zero-divisor in this ring.

2. Consider the quotient ring $S = \mathbb{R}[x, y]/(x^2 + y^2 - 1)$.

   a) (4 points) Find non-trivial zero-divisors in the quotient ring $S/(x)$.

   b) (2 points) Show that the ideal (x) is not a maximal ideal in S.

   c) (4 points) Find a maximal ideal containing (x). (You cannot use Zorn's lemma here.)

Extra space for work:

3. We say that an ideal I of a ring R is radical if whenever $x^n \in I$ for some $x \in R$ and $0 < n \in \mathbb{Z}$ then $x \in I$.

a) (7 points) Show that I is radical if and only if R/I has no non-trivial nilpotent elements. (An element y of a ring S is nilpotent if there exists a positive integer n so that $y^n = 0$.)

b) (3 points) Give an example of a non-trivial nilpotent element of some commutative ring.

Extra space for work:

4.

a) (8 points) Let R be a finite commutative ring without any zero-divisors. Show that R is a field.

b) (2 points) Give an example of a commutative ring R which is not a field and it does not have any zero-divisors.

Extra space for work:

5. Recall that we say that x is idempotent in a ring R if $x^2 = x$.

a) (5 points) Show that if R is an integral domain and x is an idempotent element, then x is either 0 or 1.

b) (5 points) Show that if R is a local ring (it has exactly one maximal ideal) and x is an idempotent element, then x is either 0 or 1. (Hint: You may assume Zorn's lemma for this part.)

Extra space for work: