Name: ________________________________

Midterm Exam #1 for Math 371

Apr 26, 2016

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

- You have an hour and twenty minutes for this exam.
- Please show **ALL** your work on this exam paper. Partial credit will be awarded where appropriate.
- You may quote without proof the theorems/propositions/lemmas given during lecture as long as you state each result clearly. Points may be deducted for incorrect or missing hypotheses.
- **NO** calculators, books, notes, or computers are permitted.
- **NO** form of cheating will be tolerated. You are expected to uphold the Code of Academic Integrity.

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this midterm examination.

Name (printed): ________________________________

Signature: ________________________________

Date: ________________________________

1. Consider the field F of 16 elements.

(a) (3 points) How many Abelian groups of size 16 are up to isomorphism?

b) (3 points) Show that
$$F \cong \mathbb{Z}/2\mathbb{Z} \oplus \mathbb{Z}/2\mathbb{Z} \oplus \mathbb{Z}/2\mathbb{Z} \oplus \mathbb{Z}/2\mathbb{Z}$$
as Abelian groups, where the group structure on F is given by the addition on F.

c) (4 points) Give an example of ring of 16 elements which is not isomorphic to this field.

2. Let ζ_8 be a primitive 8-th root of unity in $\mathbb{C}$.

a) (4 points) Show that ζ_8 is a root of the polynomial $x^4 + 1 = 0$.

b) (4 points) Use Eisentstein criterion to show that $x^4 + 1$ is irreducible over $\mathbb{Q}$.

c) (2 points) What is the degree of the field extension $\mathbb{Q} \subseteq \mathbb{Q}(\zeta_8)$?

Extra space for work:

3. (10 points) Let F be a field. We know that all elements of F satisfy $x^{10} - x^2 = 0$. How many such fields exist (up to isomorphism)?

Extra space for work:

4. Let R be an integral domain and M an R-module. We say that an element $m \in M$ is torsion if there exists a non-zero element $r \in R$ so that $rm = 0$. (the 0 element of M)

a) (4 points) Show that the torsion elements form a submodule of M, let us denote it by M_t.

b) (3 points) Show that
$$(M \oplus N)_t \cong M_t \oplus N_t.$$

c) (3 points) Give an example of a commutative ring R which is not an integral domain and an R-module M so that the set of torsion elements of M is not a submodule of M.

Extra space for work:

5. (10 points) Recall that a field extension $E \subseteq F$ is algebraic for every element $s \in F$ there exists a non-zero polynomial $p(x) \in E[x]$ so that $p(s) = 0$. Recall that a field extension is finite if F is finite dimensional as a vector space over E (with natural vector space structure).

a) (7 points) Prove that any finite extension is algebraic.

b) (3 points) Give an example of a finite extension which is not Galois.

Extra space for work: