Name: ________________________________

Midterm Exam #1 for Math 503

Feb 14, 2017

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

- You have an hour and twenty minutes for this exam.
- Please show **ALL** your work on this exam paper. Partial credit will be awarded where appropriate.
- You may quote without proof the theorems/propositions/lemmas given during lecture as long as you state each result clearly. Points may be deducted for incorrect or missing hypotheses.
- **NO** calculators, books, notes, or computers are permitted.
- **NO** form of cheating will be tolerated. You are expected to uphold the Code of Academic Integrity.

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this midterm examination.

Name (printed): ________________________________

Signature: ________________________________

Date: ________________________________

1. Consider the set $\{\frac{a}{b} : a, b \in \mathbb{Z},\ \text{and } b \text{ is a power of } 2\}$.

   a) (4 points) Show that this is a ring where the addition and multiplication are given by addition and multiplication of rational numbers.

   b) (3 points) Show that the ideal generated by 2 is the whole ring.

   (c) (3 points) Show that the ideal generate by 3 is a maximal ideal.

2. (10 points) Which of the following is an integral domain? Justify your answer.

   1. $\mathbb{Q}[x]/(x^2+1)$
   2. $\mathbb{R}[x]/(x^2+1)$
   3. $\mathbb{C}[x]/(x^2+1)$

Extra space for work:

3. (10 points) Find the minimal polynomial of $i + 3$ over $\mathbb{Q}$.

Extra space for work:

4. (10 points) We say that an element x in a ring R is iprime if x is not 0 or a unit and whenever $x|ab$ holds in R, then $x|a$ or $x|b$. Recall that an element x in a ring R is irreducible if x is not 0 or a unit and whenever $x = ab$ holds in R, then a or b is a unit.

Find all irreducible elements and all prime elements in $\mathbb{Z}/12\mathbb{Z}$.

Extra space for work:

5. Consider the infinite-by-infinite real matrices (parametrized by positive integers) so that every row has only finitely many non-zero entries (we will denote this set by $\mathbb{R}_r^{\infty\times\infty}$). For instance this is such a matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{pmatrix}$$

a) (3 points) Show that this is a ring with the usual matrix addition and matrix multiplication.

b) (4 points) Let U be a subspace of the infinite dimensional vector space generated by $(1, 0, 0, \ldots)$, $(0, 1, 0, \ldots)$... Show that

$$\{M \in \mathbb{R}_r^{\infty\times\infty} | \text{every row of } M \text{ is in } U\}$$

form a left ideal.

c) (3 points) Show that $\mathbb{R}_r^{\infty\times\infty}$ does not satisfy ACC for left-ideals.

Extra space for work: