Name: ________________________________

Midterm Exam #2 for Math 503

Apr 25, 2017

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

- You have an hour and twenty minutes for this exam.
- Please show **ALL** your work on this exam paper. Partial credit will be awarded where appropriate.
- You may quote without proof the theorems/propositions/lemmas given during lecture as long as you state each result clearly. Points may be deducted for incorrect or missing hypotheses.
- **NO** calculators, books, notes, or computers are permitted.
- **NO** form of cheating will be tolerated. You are expected to uphold the Code of Academic Integrity.

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this midterm examination.

Name (printed): ________________________________

Signature: ________________________________

Date: ________________________________

1.

a) (5 points) Give an example of a degree 4 field extension of $\mathbb{Q}$ which is not Galois.

b) (5 points) Give an example of a degree 4 field extension of $\mathbb{Q}$ which is Galois.

Justify your answers as well.

2. Let A be an Abelian group of order 30 and B be an Abelian group of order 4. Is it possible that

$$A \otimes_{\mathbb{Z}} B$$

is of order 8?

Extra space for work:

3. (10 points) Let F be a field. We know that all elements of F satisfy $x^{10} - x^2 = 0$. How many such fields exist (up to isomorphism)?

Extra space for work:

4. How many covariant functors exist $C \to C$, where C is the following category:

- It has 2 objects, denote them x and y.
- The sets $Hom(x, x)$, $Hom(x, y)$, $Hom(y, y)$ are all of size 1 and $Hom(y, x)$ is empty?

Extra space for work:

5. Consider the group ring $\mathbb{R}[\mathbb{Z}/3\mathbb{Z}]$.

- (6 points) Use Maschke's theorem to show that $\mathbb{R}[\mathbb{Z}/3\mathbb{Z}]$ is either isomorphic to $\mathbb{R} \oplus \mathbb{C}$ or $\mathbb{R} \oplus \mathbb{R} \oplus \mathbb{R}$ as rings.
- (3 points) Let M be the 1-dimensional $\mathbb{R}$ vector space generated by an element $a\bar{0} + b\bar{1} + c\bar{2}$ of the group rings. Show that M is a submodule of $\mathbb{R}[\mathbb{Z}/3\mathbb{Z}]$ as a left $\mathbb{R}[\mathbb{Z}/3\mathbb{Z}]$-module iff $a = b = c$.
- (1 point) Conclude that $\mathbb{R}[\mathbb{Z}/3\mathbb{Z}]$ is isomorphic to $\mathbb{R} \oplus \mathbb{C}$ as rings.

Extra space for work: