Math 103 (2004 Summer Session II)
Final Exam

Instructions: Time: 2 hours. No calculators of any description allowed. Please write down all necessary steps that lead to your solution. Total points available: 110. Answer each question on separate sheets. Write your name on every sheet that you use.

1. Find the following limits: (5 pts each)

   (i) $\lim_{x\to 2}(x^4 - 3x^2 + 5x - 8)$

   (ii) $\lim_{x\to 3}\left(\dfrac{x^2 - x - 6}{x^2 - 4x + 3}\right)$

   (iii) $\lim_{x\to 1}\left(\dfrac{\sqrt{x+3} - 2}{x - 1}\right)$

   (iv) $\lim_{x\to 2^-}(x^2+1)\dfrac{|x-2|}{x-2}$

   (v) $\lim_{x\to\infty}\left(\dfrac{x^3 + 3x^2 - 2x + 4}{x^4 + x^2 - x + 1}\right)$

2. For what values of a is

$$g(x) = \begin{cases} x^3 + 1, & x \geq 1 \\ ax + \frac{1}{2}, & x < 1 \end{cases}$$

   continuous for all x? (10 pts)

3. Using the limit definition, find the derivative of $y = x^2 + 2$. (5 pts)

4. For each of the following functions, find the indicated derivative: (5 pts each)

   (i) $y = x^4 + 3x^2 - 4;\quad \frac{dy}{dx}$

   (ii) $y = (\sin\theta + \cos\theta)\sec\theta;\quad \frac{dy}{d\theta}$

   (iii) $f(t) = \frac{t^2+1}{t^2+2};\quad f'(t)$

   (iv) $y = (1 - \sin(2t))^{\frac{1}{3}};\quad \frac{dy}{dt}$

   (v) $x^2y + y^2 - x = 2;\quad \frac{dy}{dx}$

   (vi) $y = e^{4(\ln x)^2};\quad \frac{dy}{dx}$

   (Hint: In part (vi), recall that $\frac{d}{dx}e^x = e^x$ and $\frac{d}{dx}\ln x = \frac{1}{x}$.)

5. (i) Find the intervals on which the function $f(x) = -\frac{1}{3}x^3 - x^2 + 7$ is increasing and decreasing. (6 pts)
(ii) Find all local maxima and minima of f (if they exist) and say where they are assumed. (6 pts)
(iii) Which, if any, of the extrema are absolute? (3 pts)

6. Evaluate the following integrals: (5 pts each)

(i) $\int (x^3 + \frac{1}{2}x^2 + 2x)\ dx$

(ii) $\int (x^{\frac{5}{2}} - x^{\frac{4}{3}} + x^{-\frac{1}{7}})\ dx$

(iii) $\int x \sin(x^2)\ dx$

(iv) $\int_1^4 \left(\frac{\sqrt{x}}{x} + \frac{2}{\sqrt{x}} \right)\ dx$

(v) $\int_{-1}^1 \frac{2t^2\ dt}{\sqrt[3]{t^3-4}}$