

Math 114, solutions to Assignment 10

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These are the solutions to the tenth homework assignment.

1 Section 16.8, #38.

We can write this integral as

$$\lim_{a \rightarrow \infty} \iiint_{S_a} \sqrt{x^2 + y^2 + z^2} e^{-(x^2 + y^2 + z^2)} dV$$

where S_a is the sphere of radius a centered at the origin. Converting to spherical coordinates gives

$$\lim_{a \rightarrow \infty} \int_0^\pi \int_0^{2\pi} \int_0^a (\rho e^{-\rho^2}) \rho^2 \sin \phi \, d\rho \, d\theta \, d\phi.$$

The integral can be factored to give

$$\lim_{a \rightarrow \infty} \left(\int_0^\pi \sin \phi \, d\phi \right) \left(\int_0^{2\pi} d\theta \right) \left(\int_0^a \rho^3 e^{-\rho^2} \, d\rho \right).$$

The first two integrals are 2 and 2π , respectively; pulling these out of the limit we get

$$4\pi \lim_{a \rightarrow \infty} \int_0^a \rho^3 e^{-\rho^2} \, d\rho.$$

Let $t = \rho^2$; then $dt = 2\rho \, d\rho$, and this is

$$4\pi \lim_{a \rightarrow \infty} \int_0^a \frac{1}{2} t e^{-t} \, dt.$$

Pulling out the constant $1/2$, we get

$$2\pi \lim_{a \rightarrow \infty} \int_0^a t e^{-t} \, dt.$$

We can do this integral by parts, letting $u = t, dv = e^{-t} dt$; thus we have $du = dt, v = -e^{-t}$. This gives

$$2\pi \lim_{a \rightarrow \infty} \left[\frac{-t}{e^t} \Big|_{t=0}^a + \int_0^a e^{-t} \, dt \right].$$

The term $\frac{-t}{e^t} \Big|_{t=0}^a$ is equal to zero (apply L'Hopital's rule) and thus we have

$$2\pi \lim_{a \rightarrow \infty} \int_0^a e^{-t} dt.$$

Integrating, this is

$$2\pi \lim_{a \rightarrow \infty} (1 - e^{-a})$$

which is just 2π . Thus the original integral is 2π .