Mathematics 170: Ideas in Mathematics

Homework 1

This assignment is due Thursday, May 28, 2009, at the beginning of class. Please hand the homework in in class. If you can't make it to class, place it in my mailbox in the departmental office (DRL 4W1) or bring it to my office (DRL 4N27). You are allowed to talk about the homework with each other, but please write it up alone.

1. Why are you taking this course? Please write a paragraph or so explaining the previous experience you have with mathematics and what you hope to get out of this class. I'm looking for something more than "I need to take this class to fulfill a requirement" – there are many courses satisfying the distribution requirements, so why this one?

2. Consider seventeen points in a square with side length 1. Given these points, you could compute the shortest distance between any pair of them. Give an upper bound for this distance analogous to the one given in class.

3. Consider fifteen points arranged in an equilateral triangle. (You should have in mind the rack that holds the balls at the beginning of a game of pool.) Prove that if you color each of these points either black or white, there will be an equilateral triangle with its three corners all the same color. You could in theory just show all the colorings, but there are too many! I'm looking for a well-organized argument.

4. Recall that if m and n are relatively prime (that is, they have no common factor) and exactly one of them is even, then $(m^2 - n^2, 2mn, m^2 + n^2)$ form a primitive Pythagorean triple. List all the primitive Pythagorean triples with all three elements less than 100. (For example, $(65, 72, 97)$ should be on your list, but $(91, 60, 109)$ should not since $109 > 100$.)

5. Prove by induction that $1^2 + 2^2 + ... + n^2 = n(n+1)(2n+1)/6$.

6. We saw in class that the continued fraction expansion of $\sqrt{2}$ is $[1, 2, 2, 2, 2, \ldots]$. Find the continued fraction expansion of $\sqrt{5}$ and of $\sqrt{10}$. Make a conjecture about the continued fraction expansion of $\sqrt{n^2+1}$, where n is an integer.

7. (a) Determine the first 10 terms of the sequence $a_0 = 1$, $a_1 = 2$, $a_n = a_{n-1} + 2a_{n-2}$ for $n \geq 2$, and conjecture a formula for a_n. (b) Determine the first 10 terms of the sequence $b_0 = 1$, $b_1 = -1$, $b_n = b_{n-1} + 2b_{n-2}$ for $n \geq 2$, and conjecture a formula for b_n. (c) Determine the first 10 terms of the sequence $c_0 = 2$, $c_1 = 1$, $c_n = c_{n-1} + 2c_{n-2}$ for $n \geq 2$, and conjecture a formula for c_n. How does this sequence compare to the previous two?

8. Give a table of values of $(F_{n+1})^2 + (F_n)^2$ for $n = 1, 2, \ldots, 10$ where F_n is the nth Fibonacci number. Conjecture a simple formula for $(F_{n+1})^2 + (F_n)^2$. (Extra credit: prove your formula is correct.)

9. The Golden Ratio $(1+\sqrt{5})/2$ occurs often in mathematics and in natural phenomena; it is often alleged to occur in human phenomena (visual arts, music, architecture, etc.) Give an example of somewhere where the Golden Ratio occurs or is alleged to occur (the Internet is full of them!); provide a photocopy or a Web link. Explain whether you believe the Golden Ratio "actually" occurs here or not.