Mathematics 170: Ideas in Mathematics

Homework 2

This assignment is due Tuesday, June 2, 2009, at the beginning of class. Please hand the homework in in class. If you can't make it to class, place it in my mailbox in the departmental office (DRL 4W1) or bring it to my office (DRL 4N27). You are allowed to talk about the homework with each other, but please write it up alone.

1. Make a list of the prime numbers up to 200. You should do this yourself. Feel free to check your list against any of the many lists available online before going on to the remaining questions. There should be 46 numbers in your list.

2. How many of the numbers on your list are:
(a) Congruent to 1 mod 3?
(b) Congruent to 2 mod 3?
(c) Congruent to 1 mod 4?
(d) Congruent to 2 mod 4?
(e) Congruent to 3 mod 4?

3. What do you notice about the numbers in Problem 2? In particular, what do you think would happen if:
(a) we had replaced 200 with some larger number in Problem 1?
(b) we looked at the remainders when dividing by some larger number?

4. Estimate the number of prime numbers which are less than 10^{10}. (B+S 2.3.17)

5. We saw twin primes, pairs of numbers $n, n+2$ which are both prime – for example the pairs $(3, 5)$ or $(17, 19)$. Is it possible for three numbers $n, n+2$, and $n+4$ to all be prime? If so, give an example. If not, why not?

6. Let A and B be two natural numbers. Suppose that when A is divided by n, the remainder is a, and when B is divided by n, the remainder is b. How does the remainder when $A + B$ is divided by n compare with the remainder when $a + b$ is divided by n? (B+S 2.3.33)

7. Find the missing digit (the digit that replaces the X) in the following UPC codes: 02850011070X, 015839X00015, 070X72366718. (B+S 2.4.14; B+S 2.4.17; the third one is for a folder they (used to?) sell at the Penn Bookstore.)

8. In the UPC, why is 3 the number every other digit is multiplied by rather than 6? (Hint: multiply every digit from 0 to 9 by 3 and look at the answers mod 10. Do the same with 6 and compare your results.) Are there other numbers besides 3 that would function effectively? What number would you try? (B+S 2.4.38) (Bonus: what if we had twelve fingers, and used twelve digits instead of ten?)

9. For each number n from 1 to 4, compute $n^2 \bmod 5$, $n^3 \bmod 5$, and $n^4 \bmod 5$. Do you notice anything surprising? (B+S 2.4.39)