Mathematics 170: Ideas in Mathematics

Homework 4

This assignment is due Wednesday, June 10, 2009, at the beginning of class. Please hand the homework in in class. If you can't make it to class, place it in my mailbox in the departmental office (DRL 4W1) or bring it to my office (DRL 4N27). You are allowed to talk about the homework with each other, but please write it up alone.

Remember that the midterm exam is on Thursday, June 11. If you hand in any part of the homework on Tuesday, I'll grade it and get it back to you on Wednesday. However, if you hand in a solution to a problem on Tuesday, you may *not* hand in a revised solution to it on Wednesday.

1. Express the number $5.63121212\ldots$ as a fraction:
(a) by expressing some integer multiple of it as a terminating decimal, and dividing;
(b) by summing a geometric series.

2. By long division, find the decimal expansion of $2/13$. This is a repeating expansion; explain why you stop doing the long division when you do.

3. Show that the number $0.246810121416182022\ldots$, constructed by writing down each of the even numbers in order after the decimal point, is irrational. (B+S 2.7.27)

4. Show that between any two real numbers (either rational or irrational), we can always find a rational number. (This property has a formal name: the rational numbers are "dense in the reals".) You should clearly explain how, if presented with two real numbers, you could write down a rational number between them. (B+S 2.7.37)

5. Show that the cardinality of the set of natural numbers $\{1, 2, 3, \ldots\}$ and the cardinality of the set of even numbers $\{2, 4, 6, \ldots\}$ are equal by describing an explicit one-to-one correspondence. (B+S 3.2.7)

6. Suppose you have infinitely many bowling balls and two huge barrels. You take the bowling balls and put each ball in one of the two barrels. What can you conclude about the cardinality of at least one of the barrels? Prove your answer. (B+S 3.2.28)

7. Is the cardinality of the set of all *triplets* of natural numbers the same as the cardinality of the natural numbers? That is, can we find a one-to-one correspondence between triplets (a, b, c) where a, b, c are all natural numbers, and natural numbers themselves; or, can we write a list of these triplets that contains all of them? If yes, explain how you would construct such a list. If no, why is this impossible? (Bonus: explain how you would generalize your answer to lists of n natural numbers, for an arbitrary positive integer n.)

8. Consider a set of countably many circles. Suppose you have two markers, one red and one blue, and you color each circle one of the two colors. How many ways can you color the circles? Do you think that the set of all possible circle colorings has the same cardinality as the set of all natural numbers? Make a guess and explain it; you don't need to rigorously justify your answer. (B+S 3.2.35)