Mathematics 170: Ideas in Mathematics

Homework 6

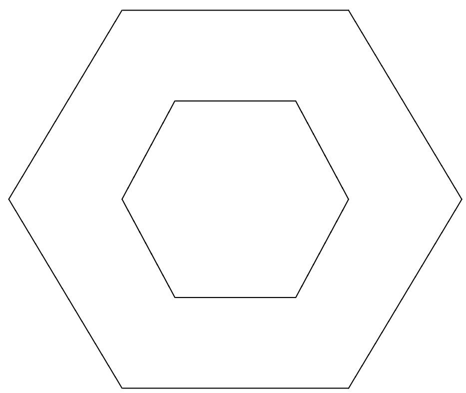
This assignment is due Monday, June 22, 2009, at the beginning of class. Please hand the homework in in class. If you can't make it to class, place it in my mailbox in the departmental office (DRL 4W1) or bring it to my office (DRL 4N27). You are allowed to talk about the homework with each other, but please write it up alone.

1. Consider the closed polygonal curve at the *left* in Section 4.2, Problem 7.

(a) According to the Art Gallery Theorem, how many guards (stationary, in swiveling chairs) will be required to guard the entire gallery?

(b) Triangulate the gallery and show where the guards must be placed.

2. Consider the gallery illustrated below:



Note that the hexagonal "hole" in the middle is not part of the gallery.

(a) Split up the gallery into two simple closed polygonal regions. (There is more than one way to do this.)

(b) For each of the regions in part (a), use the procedure from the Art Gallery Theorem to position guards so that they guard that region.

(c) Combine your answers from (b) to show how to position guards to guard the entire region.

3. Draw a picture illustrating the fact that, although we can place 2 regular pentagons (5-sided polygons) and a regular decagon (10-sided polygon) around a single vertex, we cannot extend this to a tessellation of the place which has this arrangement at every vertex.

4. Consider the truncated cube, which is obtained from the ordinary cube by cutting off each corner. (See the picture on p. 370 of the text, or on Wikipedia.)

(a) How many faces does it have? Edges? Vertices? Your answer should refer back to the (ordinary) cube to make the counting easier; recall that the ordinary cube has 6 faces, 12 edges, and 8 vertices.

(b) How many rotational symmetries does it have? Justify your answer.

(c) Your answer to (b) should be a number we've seen before in a similar context. Where have we seen this number before, and why are we seeing it again?

5. Take a cube. Put a point in the middle of each face. Now draw straight lines to the middles of each of the sides of that fact, producing a plus sign + on each face. The kinked line that goes from the center of one face to the center of an adjacent face forms a bent edge on this cubical world. Thus we have created eight "bent" triangles whose vertices are the centers of the faces of the cube.

(a) What is the sum of the angles for each of those triangles?

(b) What is the sum of all the angles of all the triangles, and how does this compare to the sum of all the angles of eight ordinary triangles in the plane? (B+S 4.6.32)

6. Repeat problem 5 for either the tetrahedron or the dodecahedron. (B+S 4.6.33 or 34)

7. We saw how to build cubes in all dimensions; how about triangles? A 0-dimensional triangle is just a point. A 1-dimensional triangle is a line segment; you know what a 2-dimensional triangle looks like; a 3-dimensional triangle is a tetrahedron. What is the pattern? We take the triangle we just created and then add a new point in the next dimension "above" the triangle. If we draw new edges from the vertices of the triangles to our new point, then we have a triangle one dimension higher.

(a) Sketch a 4-dimensional "triangle" and a 5-dimensional "triangle".

(b) Fill in a table which shows the number of vertices, edges, 2-dimensional faces, and 3-dimensional faces for the "triangles" in 1, 2, 3, 4, and 5 dimensions.

(c) Guess formulas which give the number of vertices, edges, 2-dimensional faces, and 3-dimensional faces for an n-dimensional "triangle".

(B+S 4.7.16)