Mathematics 170: Ideas in Mathematics

Homework 7

This assignment is due Thursday, June 25, 2009, at the beginning of class. Please hand the homework in in class. If you can't make it to class, place it in my mailbox in the departmental office (DRL 4W1) or bring it to my office (DRL 4N27). You are allowed to talk about the homework with each other, but please write it up alone.

1. (This is a repeat of problem 7 on HW 6.) We saw how to build cubes in all dimensions; how about triangles? A 0-dimensional triangle is just a point. A 1-dimensional triangle is a line segment; you know what a 2-dimensional triangle looks like; a 3-dimensional triangle is a tetrahedron. What is the pattern? We take the triangle we just created and then add a new point in the next dimension "above" the triangle. If we draw new edges from the vertices of the triangles to our new point, then we have a triangle one dimension higher.

(a) Sketch a 4-dimensional "triangle" and a 5-dimensional "triangle".

(b) Fill in a table which shows the number of vertices, edges, 2-dimensional faces, and 3-dimensional faces for the "triangles" in 1, 2, 3, 4, and 5 dimensions.

(c) Guess formulas which give the number of vertices, edges, 2-dimensional faces, and 3-dimensional faces for an n-dimensional "triangle". (B+S 4.7.16)

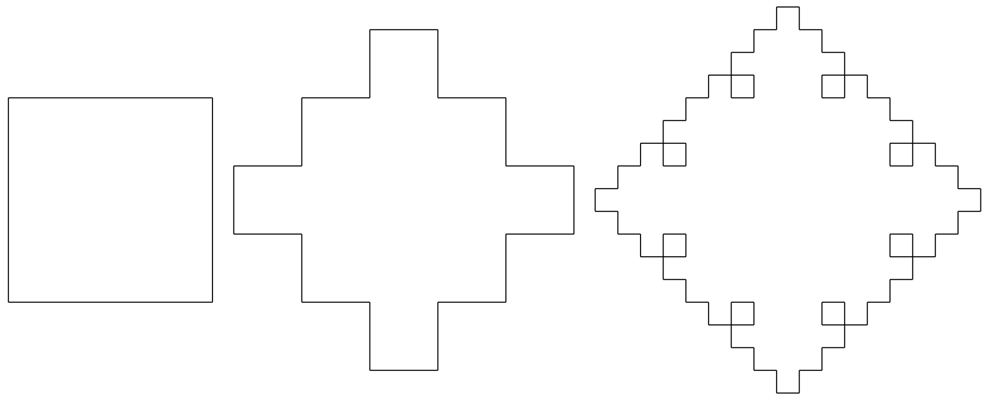
2. Consider the five regular solids with their vertices cut off. These objects are called *truncated solids*. For each truncated solid, count the number of vertices, edges, and faces, and verify that the Euler characteristic is correct. (See p. 370 of the text for pictures. Also, note that the things you should be "counting" here include those vertices, edges, and faces which are hidden in the drawings.) (B+S 5.3.22)

3. Three hollowed, triangular prisms are used to make a torus. Carefully count the number of vertices, faces, and edges for this prismatic torus. Compute the Euler characteristic for this torus. (B+S 5.3.29)

4. Carefully count the number of vertices, faces, and edges for a two-holed torus. One way to view this two-holed torus is as two copies of the torus in problem 3, with one side removed from each and then the open edges glued together. This operation is called the *connected sum*. Compute the Euler characteristic for this two-holed torus. (B+S 5.3.30)

5. Recall that in class we constructed the Koch snowflake, a shape which had finite area but infinite perimeter, by starting with an equilateral triangle, replacing each edge with four edges of one-third the length in a certain way, and repeating this process infinitely many times. (See p. 435 of the text for a picture of what happens to *one* edge when we do this.)

We can construct a similar "square Koch snowflake" by beginning with a square and replacing each edge with *five* edges of one-third the length, such that the three middle edges form a square. (Note that the boundary of this snowflake actually touches itself at various points.) The first few iterations of this appear as follows.



Assume that the original square has side length 1.

(a) What is the length of the nth curve in this series? (Note that the nth curve in this series is made up of a large number of short segments of the same length; you should find the length of each short segment and the number of them.)

(b) What is the area enclosed by the nth curve in this series? (Note that the nth curve in this series is obtained from the $(n-1)$st curve by adding a large number of small squares. For example, we obtain the second curve from the first one by adding four squares, each of which has side length $1/3$ and thus area $1/9$. So the area enclosed by the second curve is $1 + 4/9$.) Your answer should involve summing a geometric series.

6. Start with the point 0. Flip a coin. If it comes up heads, move $2/3$ of the way toward 1, if it comes up tails, move $2/3$ of the way to 0 from wherever you are at the time. Repeat forever. The points you find are drawing a picture of the Cantor set. Verify that any point in the Cantor set will move to another point in the Cantor set under the coin-flipping-and-moving process. (B+S 6.3.26)