Mathematics 170: Ideas in Mathematics

Midterm study guide

The midterm exam is on Thursday, June 11, from 4:20 to 5:55 pm. I plan to start and end promptly.

There will be about ten questions. As you might expect, they'll be shorter than the homework questions. Each problem will carry approximately equal weight; I'll explicitly indicate the weights on the exam.

Some of the questions (about six) will ask you to compute things or give simple proofs – for example, I might ask you to prove that a certain infinite set is countable, or that a certain number is irrational. For the purely computational problems, I intend to keep the arithmetic reasonably simple. You can bring a calculator; it won't be 100 percent necessary, but it might help. Due to the time-limited nature of the exam, I won't require as much explanation as I would on the homework.

Proofs that require a lot of algebra aren't something I'll ask you to reproduce on the test – so, for example, the proof that numbers have unique factorizations into primes would be too difficult, as would the proofs involving the exact formula for the Fibonacci numbers. Similarly, I won't be asking for extensive computations – the RSA cryptography computations can take a while, so I won't ask you to do them. There may be a problem that asks you to prove by induction that a certain sum holds.

The other questions (about four) will ask for short written answers – probably a few sentences each. For example, I might ask you to explain the major steps of one of the longer proofs we've done, or to There is no need to write anything very long here; quality is more important than quantity.

I'll devote the final portion of Wednesday's class (somewhere between fifteen minutes and half an hour) to your questions. I'll also hold my usual Wednesday office hour immediately following class.

Here is a list of topics you should concentrate on, roughly in the order we covered them.

- Pigeonhole principle
- Proofs by induction
- Fibonacci numbers
- Prime factorization
- Finding the primes: the sieve of Eratosthenes
- Proof that there are infinitely many primes
- Proof that prime factorizations are unique. (Why does this proof work?)
- Modular arithmetic
- Calculation of UPCs
- Fermat's little theorem, Euler's theorem
- Exponentiation by squaring, the RSA algorithm
- Rational and irrational numbers
- Decimal expansions of rational numbers

- Proofs that various sets (rationals, etc.) are countable
- The proof that there are more real numbers than natural numbers.

This list corresponds fairly closely with material that was on the homework; thus making sure you understand how to solve the homework problems will be one of the best ways to study for the exam.

Here are some things that I spent a fair amount of time on in class but explicitly will *not* test.

- Anything from the first half or so of the first day's lecture; that was meant to tell a story about "what mathematics is".
- Pythagorean triplets
- The exact formula for the Fibonacci numbers, $F_n = \frac{1}{\sqrt{5}}(\phi^n - \tau^n)$.
- The Riemann hypothesis. (But you *should* be familiar with the prime number theorem.)
- The $P = NP$ problem.
- Algebraic numbers (that is, numbers that are defined as roots of polynomials) and complex numbers.
- The extended Euclidean algorithm. Any case where you might use it will be doable by trial and error.
- The Farey fractions
- Partitions of integers
- The Calkin-Wilf tree.
- Anything "historical" in nature; this is a course in math, not the history of math.