Mathematics 170: Ideas in Mathematics

Instructor: Michael Lugo, mlugo@math.upenn.edu

Dates and times: Monday, Tuesday, Wednesday, Thursday, 4:20 - 5:55 pm. The first meeting of the course is Tuesday, May 26 (Monday is Memorial Day); the last meeting is Thursday, July 2.

Location: David Rittenhouse Laboratories (DRL), room 3C4.

Office hours: Monday 3-4 and Wednesday 6-7 (that's before class on Monday and after class on Wednesday), or by appointment.

Textbook: Edward Burger and Michael Starbird, *The Heart of Mathematics: An Introduction to Effective Thinking*, Wiley, second edition. Available at the Penn Bookstore, 36th and Walnut. Some additional readings may be assigned; these will be distributed in class or will be available online. I have more to say about the role of the textbook later.

Course web site: `www.math.upenn.edu/~mlugo/170-09B`. Please check the course web site regularly; I will use it for making announcements to the entire class, instead of sending e-mails to the class as a whole. In particular, this is where you'll be able to find the homework assignments.

Prerequisites: The registrar lists no prerequisites for this course. I will assume that you know algebra and geometry as taught in high school. More important than any formal mathematical prerequisite is the ability to think.

Course description: The official course description, which you saw when you signed up for the class, is as follows.

> Natural Science & Mathematics Sector. Class of 2010 and beyond. Staff. May also be counted toward the General Requirement in Natural Science & Mathematics. Topics from among the following: logic, sets, calculus, probability, history and philosophy of mathematics, game theory, geometry, and their relevance to contemporary science and society.

This description doesn't tell you that much, though. Traditionally this course has been taught in many different ways, depending on the instructor. I see the role of this course as twofold. One goal of the course is to expose you to some of the major ideas of classical and modern mathematics. Mathematics is one of the oldest areas of human thought – some of the results we'll talk about go back as far as the ancient Greeks – and by doing mathematics we stand on the shoulders of giants. At the same time mathematics is still vital and mathematicians continue to produce new results. Although many of these results are highly technical, some can be explained in relatively simple terms and we will talk about some of these. The course is intended to give you knowledge and appreciation of these ideas.

Second, and perhaps more importantly, the course is meant to teach you about mathematical problem solving. Mathematics is a body of facts, some of which we will see, but more importantly it is a way of thinking systematically about problems. We will spend the majority of our time in class solving and talking about how to solve such problems, and you will be asked to solve such problems on homework and in exams. For many of you, especially those for whom this is your last math class, it is unlikely that the specific mathematical topics we will talk about will be used in the future; more important are the habits of organized and systematic thoughts that mathematics instills.

Grading: 25% midterm, 30% final, 35% homework, 10% participation.

Exams: There will be a midterm exam and a final exam. The midterm exam will be on Thursday, June 11, in class; the final exam will be on Thursday, July 2, in class. Some problems on the exams will be computational in nature, asking you to solve a problem like those we have done in class or on homework; some will ask you for short written answers, asking you to show that you have thought about the context of material covered in class.

The midterm exam will for the most part cover material up to Tuesday, June 9, although I reserve the right to include material from the June 10 class if we get a bit behind on June 9. The final exam is "only incidentally cumulative"; it will cover material from June 10 through June 30. (But in some places mathematics is by its nature cumulative, so don't forget everything you learned in the first half of the course after the midterm.)

Homework: I will assign approximately eight sets of homework problems during the summer session. The homework problems will be more difficult than the exam problems, since you have more time to do them. This is your opportunity to show that you can engage with the course material on a deeper level. Some of our homework problems will come from the textbook; some will come from other sources.

Homework will be due in class. If you can't make it to class, please hand in the homework *before* class begins; bring it to my office (DRL 4N27) or place it in my mailbox in the departmental office (DRL 4W1).

You may discuss the homework with your fellow students (or with anybody else). However, when you sit down to actually write up the homework, please **do it alone**.

Once during the summer session, you may hand in your homework at most one day late. To be more specific, you must hand it in as mentioned above by *4:20 pm* the day after it is due. Your grade will not be penalized. The reason I grant only a one-day grace period here is because I would like to hand back the graded homework the next day if possible.

In addition, I will drop the lowest homework grade. If you do all but one of the homeworks, I'll drop the one you didn't do.

In case of extenuating circumstances (illness, death in the family, etc.) I may make exceptions to these rules; please let me know about such circumstances as soon as you are aware of them.

Attendance: I have no formal attendance policy. However, you should come to class. There are subjects I plan to cover that are not in our textbook, and many of the homework problems will be taken from such topics. Furthermore, you cannot participate if you don't come! If you do not attend class it is your responsibility to find out what you missed.

Participation: As stated above, a large portion of what I intend to do in this class is to teach you some of the skills of mathematical problem solving. Of course, you cannot solve problems by just watching me solve problems! Thus I expect each of you to actively participate in class, by coming to class on a regular basis, asking questions, answering the questions of your fellow students, and in general engaging with the material.

Academic dishonesty: Don't do it! The work you submit in this class is expected to be your own. See Penn's Code of Academic Integrity (available at `http://www.vpul.upenn.edu/osl/acadint.html`) for further information.)

Students with disabilities: If you have a disability that requires accomodation, I should have been informed of this by the Office of Student Disabilities Services. Feel free to approach me if you have any questions.

Course calendar: This is tentative. Since I have not taught this course before, it may also be a bit overly ambitious. Section numbers refer to *The Heart of Mathematics* by Burger and Starbird. You should read the relevant sections of the text before class. Although I feel the entire book is worth reading, some of these sections are rather long and I only expect to cover part of some sections; I'll give some guidance as to what you should actually read.

Tue 26 May: introduction to course, what is mathematics?, pigeonhole principle (2.1), Fibonacci numbers (2.2)
Wed 27 May: more on Fibonacci numbers, continued fractions, tilings (2.2)
Thu 28 May: prime numbers, the Euclidean algorithm (2.3)

Mon 01 Jun: modular arithmetic, Fermat's little theorem, RSA cryptography (2.4-2.5)
Tue 02 Jun: rational and irrational numbers (2.6-2.7)
Wed 03 Jun: sizes of infinity, "recounting the rationals" (3.2)
Thu 04 Jun: Cantor's diagonal argument, Godel's theorem (3.3-3.4)

Mon 08 Jun: the infinite in geometry, dimension (3.5)
Tue 09 Jun: the Pythagorean theorem and its connections to number theory (4.1)
Wed 10 Jun: coloring things (the "art gallery theorem", coloring maps, etc.) (4.2)
Thu 11 Jun: midterm exam

Mon 15 Jun: the golden ratio in geometry, the classical unsolvable geometric problems, the Platonic solids (4.3, 4.5)
Tue 16 Jun: non-Euclidean geometry, *Flatland* (4.6, 4.7)
Wed 17 Jun: topology, knot theory (5.1)

Thu 18 Jun: fractals, cellular automata, chaos, population growth models (6.1, 6.2)

Mon 22 Jun: fractals and chaos continued (6.3)

Tue 23 Jun: introduction to probability (this and the next three days basically cover Chapter 7)

Wed 24 Jun: the birthday paradox, computing expectations, craps and blackjack strategy

Thu 25 Jun: the binomial distribution and central limit theorem, random walks, Mediocristan and Extremistan

Mon 29 Jun: some applications of probability (spam filtering, elections, randomness in evolution...)

Tue 30 Jun: game theory

Wed 01 Jul: review, student questions

Thu 02 Jul: final exam