Math 103 Selected Homework Solutions, Sections 2.2 - 2.3

Section 2.2

2. $\lim_{x\to 1^-} f(x) = 3$ means that as x approaches 1 from the left, $f(x)$ approaches 3. Similarly, $\lim_{x\to 1^+} f(x) = 7$ means that as x approaches 1 from the right, $f(x)$ approaches 7. In this situation, $\lim_{x\to 1} f(x)$ does not exist because the left- and right-hand limits are not the same.

6. *Note*: Some of you explained these really well, some just wrote the answers. For this problem, you ought to say why certain limits don't exist. I didn't deduct points for lack of explanation this time, but I might in the future.

   (a) $\lim_{x\to -2^-} g(x) = -1$

   (b) $\lim_{x\to -2^+} g(x) = 1$

   (c) $\lim_{x\to -2} g(x)$ does not exist because the left- and right-hand limits are different.

   (d) $g(-2) = 1$

   (e) $\lim_{x\to 2^-} g(x) = 1$

   (f) $\lim_{x\to 2^+} g(x) = 2$

   (g) $\lim_{x\to 2} g(x)$ does not exist because the left- and right-hand limits are different.

   (h) $g(2) = 2$

   (i) $\lim_{x\to 4^+} g(x)$ does not exist because the function does not approach any single value.

   (j) $\lim_{x\to 4^-} g(x) = 2$

   (k) $g(0)$ is undefined. A common mistake was to say that $g(0) = 0$, but that's not true!

   (l) $\lim_{x\to 0} g(x) = 0$

10. $\lim_{t\to 12^-} f(t) = 150$, and this is the minimum amount of drug in the bloodstream just before the injection at $t = 12$ hours. $\lim_{t\to 12^+} f(t) = 300$, and this is the amount of drug in the bloodstream just after the injection at $t = 12$ hours.

26. As $x \to 0$, the denominator of $\frac{x-1}{x^2(x+2)}$ goes to 0, but the numerator goes to 1, so the fraction tends toward $\pm\infty$. Since the denominator is positive near 0 and the numerator is negative, we have $\lim_{x\to 0} \frac{x-1}{x^2(x+2)} = -\infty$. *Note*: Another good way to justify your answer to a problem like this is to draw a sign chart, showing the signs of the numerator and denominator around $x = 0$.

38. Note that m_0 and c are fixed. As $v \to c^-$, the fraction $v^2/c^2 \to 1^-$, and so $(1 - v^2/c^2) \to 0^+$. Thus,

$$\lim_{v\to c^-} \frac{m_0}{\sqrt{1 - v^2/c^2}} = \infty$$

since the numerator is fixed, but the denominator goes to 0. The physics interpretation of this limit is that as the speed of an object approaches the speed of light, the mass of the object increases without bound.

Section 2.3

2. (a) $\lim_{x\to 2}[f(x)+g(x)] = 2+0 = 2$
   (b) $\lim_{x\to 1}[f(x)+g(x)]$ does not exist because the limit of $g(x)$ does not exist at $x=1$.
   (c) $\lim_{x\to 0}[f(x)g(x)] = 0$
   (d) $\lim_{x\to -1} \frac{f(x)}{g(x)}$ does not exist. We can't use the quotient law for limits because $\lim_{x\to -1} g(x) = 0$. In fact, the quotient $\frac{f(x)}{g(x)}$ grows toward ∞ as $x \to -1$ from the left, and toward $-\infty$ as $x \to -1$ from the right.
   (e) $\lim_{x\to 2} x^3 f(x) = 16$
   (f) $\lim_{x\to 1} \sqrt{3+f(x)} = \sqrt{3+1} = 2$

10. (a) The equation is true everywhere except at $x=2$.
    (b) The equation involving limits is true, since the limit as $x\to 2$ only considers what happens to a function *near* 2, not at $x=2$.

20.
$$\lim_{h\to 0} \frac{(2+h)^3-8}{h} = \lim_{h\to 0} \frac{(8+12h+6h^2+h^3)-8}{h} = \lim_{h\to 0}(12+6h+h^2) = 12$$

52.
$$\lim_{v\to c^-} L_0\sqrt{1-v^2/c^2} = L_0\sqrt{\lim_{v\to c^-}(1-v^2/c^2} = L_0\sqrt{0} = 0$$

The physics interpretation is that as the speed of an object nears the speed of light, the apparent length of the object shrinks toward zero. The left-hand limit is necessary because the speed of the object must be less than the speed of light; i.e. v must increase to c from below.