Math 103 Selected Homework Solutions, Sections 2.5 and 2.6

Section 2.5

40. For any choice of c, the function $g(x)$ will be continuous on the intervals $(-\infty, 4)$ and $(4, \infty)$. We want to pick c so that $g(x)$ is continuous at $x = 4$. That is, we want the left- and right-hand limits to agree at $x = 4$, or in mathematical terms we want:

$$\lim_{x\to 4^-} g(x) = \lim_{x\to 4^+} g(x)$$

Substituting the appropriate equations, we solve:

$$\begin{aligned}\lim_{x\to 4^-}(x^2 - c^2) &= \lim_{x\to 4^+}(cx + 20)\\ 4^2 - c^2 &= 4c + 20\\ 0 &= c^2 + 4c + 4\\ 0 &= (c+2)^2\end{aligned}$$

And so we have $c = -2$.

45. Let $f(x) = x^4 + x - 3$. Note that $f(1) = -1$ and $f(2) = 15$. Since $f(x)$ is continuous (it is a polynomial), the Intermediate Value Theorem tells us that f must cross the x-axis somewhere between $x = 1$ and $x = 2$. That is, $f(x) = 0$ has a solution for some x in the interval $(1, 2)$.

Section 2.6

18. The height (in meters) of the arrow after t seconds is given by $H(t) = 58t - 0.83t^2$. This problem was challenging for many people, but it's important to understand the concepts involved. Similar problems would be good review for the exam (look at nearby problems in the book).

    (a) The velocity after 1 second is:

$$\begin{aligned}v(1) &= \lim_{h\to 0}\frac{H(1+h) - H(1)}{h}\\ &= \lim_{h\to 0}\frac{(58(1+h) - 0.83(1+h)^2) - (58(1) + 0.83(1))}{h}\\ &= \lim_{h\to 0}\frac{h(58 - 1.66 - 0.83h)}{h} = 56.34 \text{ meters/second}\end{aligned}$$

(b) The velocity when $t = a$ seconds is given by:

$$\begin{aligned} v(a) &= \lim_{h\to 0} \frac{H(a+h) - H(a)}{h} \\ &= \lim_{h\to 0} \frac{(58(a+h) - 0.83(a+h)^2) - (58(a) + 0.83(a)^2)}{h} \\ &= \lim_{h\to 0} \frac{h(58 - 1.66a - 0.83h)}{h} = 58 - 1.66a \text{ meters/second} \end{aligned}$$

(c) The arrow hits the moon when its height above ground is zero, i.e. when $H(t) = 0$. We solve $0 = 58t - 0.83t^2$ and find the two solutions are $t = 0$ and $t = 69.88$ seconds. The solution $t = 0$ is when the arrow was fired, so the arrow hits the moon at $t = 69.88$ seconds.

(d) To find the velocity of the arrow when it hits the moon, we can use the velocity formula from part (b) with $a = 69.88$: $v(69.88) = 58 - 1.66(69.88) = -58$, or 58 meters per second downward.

24. (a) Average rates of growth:

(i) from 1992 to 1996: $\dfrac{P(1996) - P(1992)}{1996 - 1992} = \dfrac{10,152 - 10,036}{4} = \dfrac{116}{4} = 29$ thousand people per year

(ii) from 1994 to 1996: $\dfrac{P(1996) - P(1994)}{1996 - 1994} = \dfrac{10,152 - 10,109}{2} = \dfrac{43}{2} = 21.5$ thousand people per year

(iii) from 1996 to 1998: $\dfrac{P(1998) - P(1996)}{1998 - 1996} = \dfrac{10,175 - 10,152}{2} = \dfrac{23}{2} = 11.5$ thousand people per year

(b) Using the values from (ii) and (iii), we estimate the instantaneous rate of growth in 1996 to be $\dfrac{21.5 + 11.5}{2} = 16.5$ thousand people per year.