Integral Practice

In problems 1-10, evaluate the integrals:

1. $\int_1^3 (3x^2 + 5x - 1)\, dx = \left[x^3 + \frac{5}{2}x^2 - x\right]_1^3 = 44$

2. $\int_1^2 \frac{1 + x^2}{x^2}\, dx = \int_1^2 \left(x^{-2} + 1\right)\, dx = \left[-x^{-1} + x\right] = \frac{3}{2}$

3. $\int_2^5 (t^3 - \pi t^2)\, dt = \left[\frac{1}{4}t^4 - \frac{\pi}{3}t^3\right]_2^5 = \frac{609}{4} - 39\pi$

4. $\int \left(t\sqrt{t} + \frac{1}{t\sqrt{t}}\right)\, dt = \int \left(t^{3/2} + t^{-3/2}\right)\, dt = \frac{2}{5}t^{5/2} - 2t^{-1/2} + C$

5. $\int_0^{\pi/4} (\sin\theta + \cos\theta)\, d\theta = [-\cos\theta + \sin\theta]_0^{\pi/4} = 1$

6. $\int_{-3}^{-1} \frac{2}{u^3}\, du = \int_{-2}^{-1} 2u^{-3}\, du = -u^{-2}\Big|_{-3}^{-1} = -\frac{8}{9}$

7. $\int_0^{\pi/4} \frac{1}{\cos^2 x}\, dx = = \int_0^{\pi/4} \sec^2 x = \tan x\Big|_0^{\pi/4} = 1$

8. $\int_{-\pi}^{\pi} \sin\theta\, d\theta = -\cos x\Big|_{-\pi}^{\pi} = 0$

9. $\int \cos(x + 1)\, dx = \sin(x + 1) + C$

10. $\int_1^4 (t - 2)^3\, dt = \frac{1}{4}(t - 2)^4\Big|_1^4 = \frac{15}{4}$

11. Calculate the exact area between the graph of $y = 7 - 8x + x^2$ and the x-axis.

    First we find the x-intercepts of $y = 7 - 8x + x^2$, which are $x = 1$ and $x = 7$.

    $$\int_1^7 (7 - 8x + x^2)\, dx = \left[7x - 4x^2 + \frac{1}{3}x^3\right]_1^7 = -36$$

    The integral is negative because the function is below the x-axis. The area, then, is 36.

12. Find the positive value c which makes the area between the graph of $y = -x^2 + c^2$ and the x-axis equal to 36.

The x-intercepts of $y = -x^2 + c^2$ are $x = \pm c$. Since the function is above the x-axis for $-c < x < c$, the area between the function and the x-axis is:

$$\int_{-c}^{c} (-x^2 + c^2)\, dx = \left[\frac{-1}{3} x^3 + c^2 x \right]_{-c}^{c} = \frac{4}{3} c^3$$

We want the area to be 36, that is, we want $\frac{4}{3}c^3 = 36$, or $c^3 = 27$, or $c = 3$.

13. If $r(t)$ is the rate at which water flows from a pipe in gallons per minute at time t, what does $\int_0^{60} r(t)\, dt$ represent?

    $\int_0^{60} r(t)\, dt$ is the number of gallons of water that flowed from the pipe from $t = 0$ to $t = 60$ minutes.

14. If t is measured in hours and $w(t)$ is measured in watts, what are the units for $\int_0^5 w(t)\, dt$?

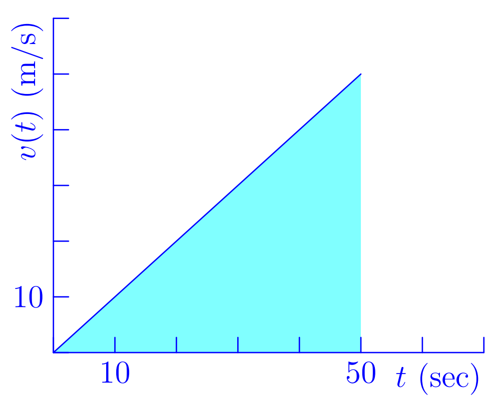
    The units of $\int_0^5 w(t)\, dt$ are *watt-hours* (i.e. watts multplied by hours). If you're interested: a *watt* is a unit of power, and a *watt-hour* is the amount of energy required to power a one-watt light bulb (or other device) for one hour.

15. A certain jet needs to reach a speed of 180 km/hr to take off. If it accelerates at 1 m/sec^2, how long of a runway does the jet need?

    A speed of 108 km/hr corresponds to 50 m/s:

$$180\ \frac{\text{km}}{\text{hr}} = \frac{180000\ \text{m}}{3600\ \text{sec}} = 50\ \frac{\text{m}}{\text{sec}}$$

If the jet accelerates at 1 m/sec^2, it takes 50 seconds to accelerate to 50 m/s. Acceleration is the derivative (slope) of the velocity function, so the graph of the velocity of accelerating jet is:



The distance the jet travels in 50 seconds is the area under the velocity function from $t = 0$ to $t = 50$, which is $\dfrac{50 \cdot 50}{2} = 1250$ meters.