

Quiz 7

Math 103 - Introduction to Calculus

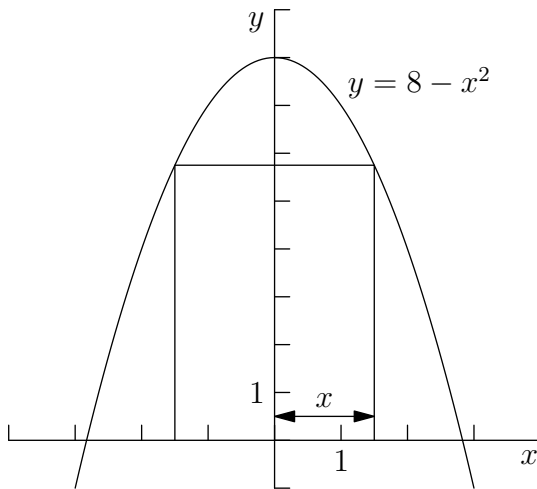
August 5, 2008

Name: _____ SOLUTION _____

Note: *In order to receive full credit, you must show work that justifies your answer.*

A rectangle has its base on the x -axis and its other two vertices above the x -axis on the parabola $y = 8 - x^2$.

- (a) Find a formula $A(x)$ for the area of the rectangle that depends only on the length x as pictured.



The width of the rectangle is $2x$, and the height is $8 - x^2$, so the area is:

$$A(x) = (2x)(8 - x^2) = 16x - 2x^3$$

- (b) Find the dimensions of the largest such rectangle.

We want to maximize the area of the rectangle.

$A'(x) = 16 - 6x^2$, so $A'(x) = 0$ when $x^2 = \frac{8}{3}$, so $x = \sqrt{\frac{8}{3}}$ (x is positive since the width of the rectangle is positive).

A sign chart confirms that $A(x)$ is in fact maximum when $x = \sqrt{\frac{8}{3}}$.

Thus, the largest such rectangle has width $2\sqrt{\frac{8}{3}} = \frac{4\sqrt{6}}{3}$ and height $8 - \frac{8}{3} = \frac{16}{3}$.