Quiz 4 Name: SOLUTIONS

Math 103 - Introduction to Calculus

July 22, 2008

Note: *In order to receive full credit, you must show work that justifies your answer.*

1. Let $f(x) = x \sin\left(\frac{1}{x}\right)$. What is $f'(x)$?

   We have to apply the product rule and the chain rule:

   $$f'(x) = 1 \cdot \sin\left(\tfrac{1}{x}\right) + x \cdot \left[\cos\left(\tfrac{1}{x}\right)\left(\tfrac{-1}{x^2}\right)\right]$$
   $$f'(x) = \sin\left(\tfrac{1}{x}\right) - \tfrac{1}{x}\cos\left(\tfrac{1}{x}\right)$$

2. Find the equation of the line tangent to $y = \sqrt{x^2 - 5}$ at $x = 3$.

   To find the equation of a line, we need a point and a slope.

   The point is $(3, y(3)) = (3, \sqrt{(3)^2 - 5}) = (3, 2)$.

   To find the slope, we differentiate using the chain rule:

   $$y' = \frac{1}{2\sqrt{x^2 - 5}} \cdot 2x$$
   $$y' = \frac{x}{\sqrt{x^2 - 5}}$$

   So the slope at $x = 3$ is $y'(3) = \dfrac{3}{\sqrt{(3)^2 - 5}} = \dfrac{3}{2}$.

   Thus, the equation of the line is $y - 2 = \frac{3}{2}(x - 3)$ or equivalently $y = \frac{3}{2}x - \frac{5}{2}$.

   Note: Some people said the equation for the line was $y - 2 = \dfrac{x}{\sqrt{x^2 - 5}}(x - 3)$, or something similar. The problem is that this is *not* the equation of a line. The fact that it contains a square root should make you very suspicious. The slope of a line should be a *number*, not $\dfrac{x}{\sqrt{x^2 - 5}}$. To obtain the correct equation for the line, evaluate the derivative at $x = 3$ to find that the slope of the line is $\frac{3}{2}$.

   Remember, lines have equations like $y - y_1 = m(x - x_1)$ or $y = mx + b$ that only contain x to the first power.