Quiz 5 Name: SOLUTION

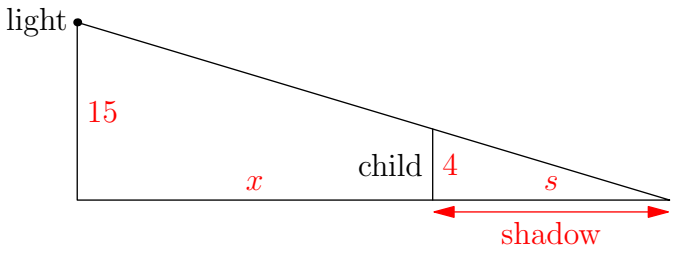
Math 103 - Introduction to Calculus

July 24, 2008

Note: *In order to receive full credit, you must show work that justifies your answer.*

A street light is mounted at the top of a pole 15 feet tall. A child 4 feet tall walks away from the pole with a speed of 2 feet per second along a straight path. How fast is the length of the child's shadow changing when the child is 10 feet from the pole? Clearly explain your solution.

Solution: Let x be the distance from the light pole to the child, and let s be the length of the child's shadow, as in the diagram:



We know: $\frac{dx}{dt} = 2$ ft/s

We want to find: $\frac{ds}{dt}$ when $x = 10$ ft

Connecting equation: by similar triangles, we have $\dfrac{x+s}{15} = \dfrac{s}{4}$. Cross-multiplying, we have $4x + 4s = 15s$, or $4x = 11s$.

Differentiate: $4\frac{dx}{dt} = 11\frac{ds}{dt}$

Substitute and solve: Since $\frac{dx}{dt} = 2$, we have $4(2) = 11\frac{ds}{dt}$, so $\frac{ds}{dt} = \frac{8}{11}$. Thus the length of the shadow is increasing at $\frac{8}{11} \approx 0.727$ feet per second.

Extension: Note that we never used the information $x = 10$. In fact this was irrelevant; the child's shadow increases at a rate of $\frac{8}{11}$ feet per second regardless of the child's distance from the light. This suggests a different way of approaching the problem, *without* calculus. Examine the triangles in the diagram: if the child is 2 feet from the light pole, the shadow length is $\frac{8}{11}$ feet. If the child is 4 feet from the light pole, its shadow length is $\frac{16}{11}$ feet. What is the length of the shadow if the child is 6 feet from the light pole? Does it make sense, just from considering the similar triangles, that the length of shadow increases by $\frac{8}{11}$ feet for each 2 feet the child moves away from the light pole?