

Quiz 6

Math 103 - Introduction to Calculus

July 29, 2008

Name: _____ SOLUTIONS _____

Note: *In order to receive full credit, you must show work that justifies your answer.*

1. Find the absolute minimum and maximum values of $f(x) = 2x^3 - 3x^2 - 12x + 3$ on the interval $[0, 3]$.

First we find the critical numbers of f .

$f'(x) = 6x^2 - 6x - 12 = 6(x - 2)(x + 1)$, so the critical numbers are $x = -1$ and $x = 2$, but only the latter is in the interval $[0, 3]$.

We compare the values of $f(x)$ at the endpoints of the interval $[0, 3]$ and at the critical number $x = 2$:

x	$f(x)$
0	3
2	-17
3	-6

From the above table, we see that the absolute minimum value is -17 , and the absolute maximum value is 3 .

2. Does there exist a function f such that $f(0) = 2$, $f(3) = -2$, and $|f'(x)| < 1$ for all x ? Either give an example or say why none exists.

If such a function exists, the Mean Value Theorem* says that for some c in the interval $(0, 3)$,

$$f'(c) = \frac{f(3) - f(0)}{3 - 0} = \frac{-2 - 2}{3} = \frac{-4}{3}.$$

But $|\frac{-4}{3}| > 1$, which violates the condition that $|f'(x)| < 1$ for all x . Thus, no such function f exists.

*The condition on f' implies that f is differentiable (and thus continuous) for all x . Therefore, the Mean Value Theorem applies.