Math 104 Selected Homework Solutions, Section 8.1

10. Let $u = \sin^{-1} x$ and $dv = dx$. Then $du = \frac{1}{\sqrt{1-x^2}}dx$ and $v = x$, so the integral becomes:

$$\int \sin^{-1} x \, dx = x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} \, dx.$$

Now we can let $t = 1 - x^2$ so $dt = -2xdx$ and we have:

$$x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} \, dx = x \sin^{-1} x + \int \frac{dt}{2\sqrt{t}} = x \sin^{-1} x + \sqrt{1-x^2} + C.$$

12. Let $u = \ln p$ and $dv = p^5 dp$:

$$\int p^5 \ln p \, dp = \frac{1}{6}p^6 \ln p - \frac{1}{6} \int p^5 \, dp = \frac{1}{6}p^6 \ln p - \frac{1}{36}p^6 + C.$$

34. First we make the substitution $t = \sqrt{x}$, so $dt = \frac{1}{2\sqrt{x}}dx$. Writing $dx = 2\sqrt{x}dt = 2tdt$, we can transform the integral:

$$\int_1^4 e^{\sqrt{x}} \, dx = \int_1^2 2te^t \, dt.$$

Now we use integration by parts. Let $u = 2t$ and $dv = e^t dt$. We have:

$$\int_1^2 2te^t \, dt = \left[2te^t\right]_1^2 - \int_1^2 2e^t \, dt = \left(4e^2 - 2e\right) - \left[2e^t\right]_1^2 = 4e^2 - 2e - 2e^2 + 2e = 2e^2.$$

52. The graphs of $y = 5 \ln x$ and $y = x \ln x$ intersect at $(1, 0)$ and $(5, 5 \ln 5)$. The area between them is:

$$\int_1^5 (5 \ln x - x \ln x) \, dx = \int_1^5 (5 - x) \ln x \, dx.$$

We can evaluate this integral using integration by parts. Let $u = \ln x$, $dv = (5 - x)dx$. Then $du = \frac{1}{x}$ and $v = 5x - \frac{1}{2}x^2$. We have:

$$\begin{aligned} \int_1^5 (5-x) \ln x dx &= \left[\left(5x - \frac{1}{2}x^2\right) \ln x\right]_1^5 - \int_1^5 \left(5 - \frac{1}{2}x\right) dx \\ &= \frac{25}{2} \ln 5 - \left[5x - \frac{1}{4}x^2\right]_1^5 = \frac{25}{2} \ln 5 - 14. \end{aligned}$$