Math 104 Selected Homework Solutions, Section 8.3 and 8.4

Section 8.4

4. Partial fraction decompositions:

   (a) Since the degree of the numerator is greater than the degree of the denominator, first use long division:

$$\frac{x^3}{x^2+4x+3} = x - 4 + \frac{13x-12}{x^2+4x+3} = x - 4 + \frac{A}{x+3} + \frac{B}{x+1}$$

   (b)

$$\frac{2x+1}{(x+1)^3(x^2+4)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3} + \frac{Dx+E}{x^2+4} + \frac{Fx+G}{(x^2+4)^2}$$

10. Using partial fraction decomposition,

$$\int \frac{1}{(t+4)(t-1)}\,dt = \int \left(\frac{-\frac{1}{5}}{t+4} + \frac{\frac{1}{5}}{t-1}\right)\,dt = -\tfrac{1}{5}\ln|t+4| + \tfrac{1}{5}\ln|t-1| + C$$

12. Using partial fraction decomposition,

$$\begin{aligned}\int_0^1 \frac{x-1}{x^2+3x+2}\,dx &= \int_0^1 \frac{x-1}{(x+1)(x+2)}\,dx = \int_0^1 \left(\frac{-2}{x+1} + \frac{3}{x+2}\right)\,dx \\ &= \left[-2\ln|x+1| + 3\ln|x+2|\right]_0^1 = (-2\ln 2 + 3\ln 3) - (0 + 3\ln 2) \\ &= 3\ln 3 - 5\ln 2 = \ln 3^3 - \ln 2^5 = \ln \tfrac{27}{32}\end{aligned}$$

Section 8.5

12. Use the substitution $u = \cos x$:

$$\int \sin x \cos(\cos x)\,dx = -\int \cos u\,du = -\sin u + C = -\sin(\cos x) + C$$

60. Factor the denominator and then use the substitution $u = x^{2/3} + 1$, so $du = \frac{2}{3}x^{-1/3}dx$:

$$\int \frac{1}{x+\sqrt[3]{x}}\,dx = \int \frac{1}{\sqrt[3]{x}(x^{2/3}+1)}\,dx = \tfrac{3}{2}\int \frac{du}{u} = \tfrac{3}{2}\ln|u| + C = \tfrac{3}{2}\ln\left|x^{2/3}+1\right| + C$$

74. First, factor the denominator to rewrite the fraction with only positive exponents. Then use the substitution $u = e^x$ and recognize the integral as number 19 in the table on page 542.

$$\int \frac{dx}{e^x - e^{-x}} = \int \frac{dx}{e^{-x}(e^{2x} - 1)} = \int \frac{e^x dx}{e^{2x} - 1} = \int \frac{du}{u^2 - 1}$$
$$= \tfrac{1}{2} \ln \left| \frac{1-u}{1+u} \right| + C = \tfrac{1}{2} \ln \left| \frac{1-e^x}{1+e^x} \right| + C$$

76. Use integration by parts twice:

$$\int \left(x^2 - bx\right) \sin 2x \, dx = -\tfrac{1}{2}\left(x^2 - bx\right) \cos 2x + \tfrac{1}{2} \int (2x - b) \cos 2x \, dx$$

let $u = x^2 - bx, dv = \sin 2x \, dx$

$$= -\tfrac{1}{2}\left(x^2 - bx\right) \cos 2x + \tfrac{1}{4}(2x - b) \sin 2x - \int \sin 2x \, dx$$

let $u = 2x - b, dv = \cos 2x \, dx$

$$= -\tfrac{1}{2}\left(x^2 - bx\right) \cos 2x + \tfrac{1}{4}(2x - b) \sin 2x + \tfrac{1}{2} \cos 2x + C$$

81. This is tricky. The solution involves integration by parts and some cancellation:

$$\int \left(2x^2 + 1\right) e^{x^2} \, dx = \int 2x^2 e^{x^2} \, dx + \int e^{x^2} \, dx$$

use integration by parts for the integral in blue, with $u = x$ and $dv = 2xe^{x^2} dx$

$$= \left[xe^{x^2} - \int e^{x^2} \, dx \right] + \int e^{x^2} \, dx$$
$$= xe^{x^2} + C$$