

## Math 104 Selected Homework Solutions, Section 8.7 and 8.8

### Section 8.7

30. The pool is measured at 7 locations, which partition it into 8 subintervals, each of width 2m. So we will estimate the area using Simpson's rule with  $n = 8$  and  $\delta x = 2$  meters.

$$\begin{aligned} A &\approx \frac{2}{3} (0 + 4(6.2) + 2(7.2) + 4(6.8) + 2(5.6) + 4(5.0) + 2(4.8) + 4(4.8) + 0) \\ &\approx \frac{2}{3} (126.4) \\ &\approx 84 \text{ meters}^2 \end{aligned}$$

### Section 8.8

2. (a)  $\int_1^2 \frac{1}{2x-1} dx$  is proper.  
(b)  $\int_0^1 \frac{1}{2x-1} dx$  is improper because  $\frac{1}{2x-1}$  has a vertical asymptote at  $x = \frac{1}{2}$ .  
(c)  $\int_{-\infty}^{\infty} \frac{\sin x}{1+x^2} dx$  is improper because the bounds are infinite.  
(d)  $\int_1^2 \ln(x-1) dx$  is improper because  $\ln(x-1)$  has a vertical asymptote at  $x = 1$ , which is a bound of integration.

6. The integral is divergent:

$$\begin{aligned} \int_{-\infty}^0 \frac{1}{2x-5} dx &= \lim_{t \rightarrow -\infty} \int_t^0 \frac{1}{2x-5} dx = \lim_{t \rightarrow -\infty} \left[ \frac{1}{2} \ln |2x-5| \right]_t^0 \\ &= \lim_{t \rightarrow -\infty} \left( \frac{1}{2} \ln(5) - \frac{1}{2} \ln |2t-5| \right) = \frac{1}{2} \ln(5) - \frac{1}{2} \lim_{t \rightarrow -\infty} \ln(2t+5) \\ &= \frac{1}{2} \ln(5) - \infty = -\infty \end{aligned}$$

10. The integral is divergent:

$$\begin{aligned} \int_{-\infty}^{-1} e^{-2t} dt &= \lim_{s \rightarrow -\infty} \int_s^{-1} e^{-2t} dt = \lim_{s \rightarrow -\infty} \left[ -\frac{1}{2} e^{-2t} \right]_s^{-1} \\ &= \lim_{s \rightarrow -\infty} \left( -\frac{1}{2} e^2 + \frac{1}{2} e^{-2s} \right) = -\frac{1}{2} e^2 + \frac{1}{2} \lim_{s \rightarrow -\infty} e^{2s} \\ &= -\frac{1}{2} e^2 + \infty = \infty \end{aligned}$$

14. The integral is divergent:

$$\begin{aligned} \int_{-\infty}^{\infty} x^2 e^{-x^3} dx &= \frac{1}{3} \int_{-\infty}^{\infty} e^{-u} du \quad \text{let } u = x^3, \text{ so } du = 3x^2 dx \\ &= \frac{1}{3} \lim_{t \rightarrow -\infty} \int_t^0 e^{-u} du + \frac{1}{3} \lim_{t \rightarrow \infty} \int_0^t e^{-u} du \\ &= \frac{1}{3} \lim_{t \rightarrow -\infty} [-e^{-u}]_t^0 + \frac{1}{3} \lim_{t \rightarrow \infty} [-e^{-u}]_0^t \\ &= \frac{1}{3} (-1 + \infty) + \frac{1}{3} (0 + 1) = \infty \end{aligned}$$

49. We compare  $\frac{\cos^2 x}{1+x^2}$  to  $\frac{1}{x^2}$ . Since  $1+x^2 > x^2$ , it follows that  $\frac{1}{1+x^2} < \frac{1}{x^2}$ . Also,  $0 \leq \cos^2 x \leq 1$ , so we have:

$$0 \leq \frac{\cos^2 x}{1+x^2} \leq \frac{1}{1+x^2} < \frac{1}{x^2}$$

The important thing is that  $0 \leq \frac{\cos^2 x}{1+x^2} \leq \frac{1}{x^2}$  for  $x \geq 1$ . We know from page 569 that  $\int_1^\infty \frac{1}{x^2} dx$  converges. Therefore, by the comparison test,  $\int_1^\infty \frac{\cos^2 x}{1+x^2} dx$  also converges.