**Quiz 9** Name: SOLUTIONS

Math 104 - Calculus I

April 23, 2008

**Note:** *In order to receive full credit, you must show work that justifies your answer.*

1. Determine whether the series converges or diverges:

$$\sum_{n=2}^{\infty} \frac{\sqrt{n}}{n-1}$$

There are several ways to show that this series diverges. A general statement such as "the series is similar to $\sum \frac{1}{\sqrt{n}}$" captures the intuition of the problem, but you should cite a specific test to make your solution rigorous. Two possible solutions are:

   (a) **Limit Comparison Test**: Comparing the given series with $\sum \frac{1}{\sqrt{n}}$, we have:

$$\lim_{n\to\infty} \frac{\frac{\sqrt{n}}{n-1}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{n}{n-1} = 1$$

   Since the limit is positive and finite, and $\sum \frac{1}{\sqrt{n}}$ diverges, the given series also diverges.

   (b) **Direct Comparison**: Notice that $\frac{1}{n-1} > \frac{1}{n}$, so $\frac{\sqrt{n}}{n-1} > \frac{\sqrt{n}}{n} = \frac{1}{\sqrt{n}}$. Then $\sum \frac{\sqrt{n}}{n-1} > \sum \frac{1}{\sqrt{n}}$, which diverges, so the given series also diverges.

2. Determine whether the series converges or diverges:

$$\sum_{n=1}^{\infty} (-1)^n \frac{3n-1}{2n+1}$$

First check to see if the terms go to zero:

$$\lim_{n\to\infty} \frac{3n-1}{2n+1} = \frac{2}{3} \neq 0$$

Since the terms of the series do not go to zero, the series diverges by the Test for Divergence.

**Note**: It is incorrect to say that the series diverges by the Alternating Series Test. The Alternating Series Test only says that a series *converges* if it satisfies certain conditions. In particular, it is incorrect to say "the terms are not strictly decreasing and therefore the series diverges." You really need the Test for Divergence here.