

Quiz 6

Math 104 - Calculus I

March 24, 2008

Name: _____

Note: *In order to receive full credit, you must show work that justifies your answer.*

1. Set up, but do not solve, an integral representing the arc length of $y = x \ln x$ from $x = 1$ to $x = 3$.

The arc length formula is: $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$. In this problem,

$$\frac{dy}{dx} = 1 \cdot \ln(x) + x \cdot \frac{1}{x} = \ln(x) + 1$$

Thus, the arc length is given by:

$$L = \int_1^3 \sqrt{1 + (\ln(x) + 1)^2} dx = \int_1^3 \sqrt{2 + 2 \ln x + (\ln x)^2} dx$$

2. Find the area of the surface obtained by rotating the graph of $y = \sqrt{x}$, $2 \leq x \leq 6$, about the x -axis.

Since $y = \sqrt{x}$, $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$. Thus, the surface area is given by:

$$\begin{aligned} S &= \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_2^6 2\pi \sqrt{x} \sqrt{1 + \frac{1}{4x}} dx \\ &= \int_2^6 2\pi \sqrt{x} \sqrt{\frac{4x+1}{4x}} dx = \int_2^6 2\pi \sqrt{\frac{x(4x+1)}{4x}} dx \\ &= \int_2^6 2\pi \sqrt{\frac{4x+1}{4}} dx = \int_2^6 2\pi \frac{\sqrt{4x+1}}{2} dx \\ &= \int_2^6 \pi \sqrt{4x+1} dx = \pi \frac{2}{3} \cdot \frac{1}{4} (4x+1)^{3/2} \Big|_2^6 \\ &= \frac{\pi}{6} (25^{3/2} - 9^{3/2}) = \frac{\pi}{6} (125 - 27) \\ &= \frac{98\pi}{6} = \frac{49\pi}{3} \end{aligned}$$