Quiz 6

Name: ______________________________

Math 104 - Calculus I
March 26, 2008

Note: *In order to receive full credit, you must show work that justifies your answer.*

1. Find the arc length of $y = \frac{2}{3}(x^2-1)^{3/2}$ from $x = 1$ to $x = 3$.

   First note that $\frac{dy}{dx} = (x^2-1)^{1/2}(2x)$. Thus,

   $$1 + \left(\tfrac{dy}{dx}\right)^2 = 1 + 4x^2(x^2-1) = 4x^4 - 4x^2 + 1 = (2x^2-1)^2$$

   The arc length is given by:

   $$L = \int_a^b \sqrt{1 + \left(\tfrac{dy}{dx}\right)^2}\, dx = \int_1^3 \sqrt{(2x-1)^2}\, dx = \int_1^3 (2x-1)\, dx$$
   $$= \left[\tfrac{2}{3}x^3 - x\right]_1^3 = (18-3) - \left(\tfrac{2}{3} - 1\right) = 15 + \tfrac{1}{3} = \tfrac{46}{3}$$

2. Set up, but do not solve, an integral representing the area of the surface obtained by rotating the graph of $y = \sin^2 x$, $0 \le x \le \frac{\pi}{2}$, about the x-axis.

   The surface area formula is: $S = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$.

   In this problem, $\frac{dy}{dx} = 2\sin x \cos x$. Thus, the surface area is given by:

   $$S = \int_0^{\pi/2} 2\pi \sin^2 x \sqrt{1 + 4\sin^2 x \cos^2 x}\, dx$$