Quiz 11

Name: ______________________________

Math 114 - Calculus II
December 3, 2008

Note: *In order to receive full credit, you must show work that justifies your answer.*

Recall the following conversion formulas:

Cartesian to polar/cylindrical	Cartesian to spherical
$x = r\cos\theta$	$x = \rho\sin\phi\cos\theta$
$y = r\cos\theta$	$y = \rho\sin\phi\sin\theta$
$(z = z)$	$z = \rho\cos\phi$

1. Evaluate the integral $\int_0^1 \int_y^{\sqrt{2-y^2}} (x+y)dxdy$ by converting to polar coordinates.

   SOLUTION: First draw the picture! The domain of integration is the wedge in the xy-plane defined by all r and θ such that $0 \le r \le \sqrt{2}$ and $0 \le \theta \le \pi/4$.
   Remembering that $dA = rdrd\theta = rd\theta dr$ in polar coordinates, we get

$$\int_0^{\sqrt{2}} \int_0^{\pi/4} (r\cos\theta + r\sin\theta) r d\theta dr = \int_0^{\sqrt{2}} r^2 \left[\int_0^{\pi/4} (\cos\theta + \sin\theta) d\theta\right] dr = \frac{2^{3/2}}{3}.$$

2. Set up, but do evaluate (unless you want to), the integral of
   $f(x,y,z) = \dfrac{(x^2+y^2+z^2)^{(e-2)/2}}{\sqrt{x^2+y^2}}$ over the solid hemisphere given by $x^2+y^2+z^2 \le 1$ and $z \ge 0$. [Hint: Do *not* use Cartisian (x,y,z) or cylindrical (r,θ,z) coordinates.]

   SOLUTION: Note that $x^2+y^2+z^2 = \rho^2$ and $\sqrt{x^2+y^2} = \rho|\sin\phi|$, which equals $\rho\sin\phi$ because $0 \le \phi \le \pi$. The solid hemisphere is parameterized by $0 \le \rho \le 1$, $0 \le \theta \le 2\pi$, and $0 \le \phi \le \pi/2$, so since the volume element in spherical coordinates is $dV = \rho^2 \sin\phi d\rho d\phi d\theta$, the integral is

$$\int_0^{2\pi} \int_0^{\pi/2} \int_0^1 \left(\frac{\rho^{e-2}}{\rho\sin\phi}\right) \left(\rho^2 \sin\phi d\rho d\phi d\theta\right),$$

   which is equal to

$$\int_0^{2\pi} \int_0^{\pi/2} \int_0^1 \rho^{e-1} d\rho d\phi d\theta = (2\pi)(\pi/2)\frac{1}{e} = \frac{\pi^2}{e}$$

Quiz 11 Name: ______________________________

Math 114 - Calculus II

December 5, 2008

Note: *In order to receive full credit, you must show work that justifies your answer.*

Recall the following conversion formulas:

Cartesian to polar/cylindrical	Cartesian to spherical
$x = r\cos\theta$	$x = \rho\sin\phi\cos\theta$
$y = r\cos\theta$	$y = \rho\sin\phi\sin\theta$
$(z = z)$	$z = \rho\cos\phi$

1. Use polar or cylindrical coordinates to find the volume bounded by the paraboloids $z = 3x^2 + 3y^2$ and $z = 4 - x^2 - y^2$.

   SOLUTION: In polar coordinates, the equations become $z = 3r^2$ and $z = 4 - r^2$. These surfaces intersect in a circle of radius R, where R satisfies $3R^2 = 4 - R^2$, or $R = 2$. Hence the volume is given by

   $$\int_0^{2\pi}\int_0^2 \left((4-r^2) - 3r^2\right) r\,dr\,d\theta = 2\pi.$$

   In cylindrical coordinates we have

   $$\int_0^{2\pi}\int_{r=0}^{r=2}\int_{z=4-r^2}^{3r^2} dV = \int_0^{2\pi}\int_0^2\int_{z=4-r^2}^{3r^2} r\,dz\,dr\,d\theta = 2\pi.$$

2. Set up, but do evaluate (unless you want to), the integral of $f(x,y,z) = \sqrt{x^2+y^2+z^2}\,e^{-(x^2+y^2+z^2)^2}$ over the solid hemisphere given by $x^2+y^2+z^2 \le 1$ and $y \ge 0$.

   SOLUTION: In spherical coordinates, $f = \rho^2 e^{-\rho^4}$, so

   $$\int_{\theta=0}^{\theta=\pi}\int_{\phi=0}^{\phi=\pi}\int_{\rho=0}^{\rho=1} f\,dV = \int_{\theta=0}^{\theta=\pi}\int_{\phi=0}^{\phi=\pi}\int_{\rho=0}^{\rho=1} \rho^2 e^{-\rho^4}\rho^2\sin\phi\,d\rho\,d\phi\,d\theta.$$