Quiz 1 Name: SOLUTIONS

Math 114 - Calculus II

September 17, 2008

Note: *In order to receive full credit, you must show work that justifies your answer.*

1. Suppose $y(x)$ satisfies the differential equation $\dfrac{dy}{dx} = (y-1)^2$. What is $\lim\limits_{x\to\infty} y(x)$?

   Separating variables, we have $\displaystyle\int \frac{dy}{(y-1)^2} = \int dx$. Integrating, $-(y-1)^{-1} = x + C$, or $y = 1 + \dfrac{1}{C-x}$.

   $$\lim_{x\to\infty} y(x) = \lim_{x\to\infty} \left(1 + \frac{1}{C-x}\right) = 1$$

2. Find the solution of $\dfrac{dy}{dt} + 2ty = y$ that satisfies the initial condition $y(1) = 5$.

   Separating variables, we solve $\displaystyle\int \frac{dy}{y} = \int (1-2t)\, dt$, so $\ln|y| = t - t^2 + C$, or $y = Ae^{t-t^2}$ where $A = \pm e^C$ is a constant.

   The initial condition $y(1) = 5$ gives $5 = Ae^0$, so $A = 5$. Thus, the solution is

   $$y = 5e^{t-t^2}.$$

Quiz 1 Name: SOLUTIONS

Math 114 - Calculus II

September 19, 2008

Note: *In order to receive full credit, you must show work that justifies your answer.*

1. Solve the differential equation $\dfrac{dy}{dx} = e^{3x+2y}$.

   Separating variables, we have $\displaystyle\int e^{-2y}\,dy = \int e^{3x}\,dx$. Integrate and solve for y:

$$\begin{aligned} -\frac{1}{2}e^{-2y} &= \frac{1}{3}e^{3x} + C \\ e^{-2y} &= -\frac{2}{3}e^{3x} + C \\ -2y &= \ln\left(-\frac{2}{3}e^{3x} + C\right) \\ y &= -\frac{1}{2}\ln\left(-\frac{2}{3}e^{3x} + C\right) \end{aligned}$$

2. Write a differential equation satisfied by the orthogonal trajectories to the family of curves $y = \dfrac{k}{x}$. (k is a constant which should *not* appear in your answer.)

   Differentiating the given equation, we have $\dfrac{dy}{dx} = \dfrac{-k}{x^2}$. Since $k = xy$, the given family of curves has slope $\dfrac{dy}{dx} = \dfrac{-y}{x}$ at the point (x, y).

   The orthogonal trajectories have slopes that are negative reciprocals of the above, so they satisfy $\dfrac{dy}{dx} = \dfrac{x}{y}$.

   Note: you weren't asked to actually find the orthogonal trajectories, but if you did, you should find them to be $y^2 - x^2 = C$.