

Quiz 2

Math 114 - Calculus II

September 24, 2008

Name: _____

Note: *In order to receive full credit, you must show work that justifies your answer.*

1. Suppose y is a function of x . Solve the following linear differential equation for y :

$$y' + \frac{y}{x} - e^x = 0.$$

SOLUTION: First rewrite this to get $y' + \frac{y}{x} = e^x$. Second, define the integrating factor

$$I(x) = e^{\int \frac{1}{x} dx} = e^{\ln x} = x.$$

Multiplying both sides by $I(x)$ and integrating, we obtain

$$xy = \int xe^x dx.$$

Using integration by parts with $u = x$ and $dv = e^x dx$, we have $du = dx$ and $v = e^x$, and hence

$$xy = xe^x - \int e^x dx = xe^x - e^x + C,$$

which implies

$$y = e^x - \frac{e^x}{x} + \frac{C}{x}.$$

2. Suppose y is a function of x . Solve the following linear differential equation for y :

$$y' + (\cos x)y = \cos x.$$

SOLUTION: Note that this equation is both separable and a first-order, linear differential equation, hence either method (separation of variables or using an integrating factor) works. To use the integrating factor, set

$$I(x) = e^{\int \cos x dx} = e^{\sin x},$$

then multiply both sides of the differential equation by $I(x)$ and integrate:

$$e^{\sin x} y = \int e^{\sin x} \cos x dx.$$

Performing a u -substitution with $u = \sin x$, we obtain

$$e^{\sin x} y = \int e^u du = e^u + C = e^{\sin x} + C,$$

and hence

$$y = 1 + Ce^{-\sin x}.$$

Quiz 2

Math 114 - Calculus II

September 26, 2008

Name: _____

Note: *In order to receive full credit, you must show work that justifies your answer.*

1. Suppose y is a function of x . Solve the following linear differential equation for y :

$$xy' + y = x \sin x.$$

SOLUTION: Divide by x to put this in the form $y' + \frac{1}{x}y = \sin x$. Then define the integrating factor:

$$I(x) = e^{\int \frac{1}{x} dx} = e^{\ln x} = x.$$

Multiplying both sides by $I(x)$ and integrating, we obtain

$$xy = \int x \sin x dx.$$

Integrating by parts with $u = x$ and $dv = \sin x dx$, we have $du = dx$ and $v = -\cos x$ and hence

$$xy = -x \cos x - \int (-\cos x) dx = -x \cos x + \sin x + C,$$

which implies

$$y = -\cos x + \frac{\sin x}{x} + \frac{C}{x}.$$

2. Suppose y is a function of x . Solve the following linear differential equation for y :

$$y' + e^x y = e^x.$$

SOLUTION: Note that this equation is separable and it's a first-order, linear differential equation; hence you could use separation of variables *or* an integrating factor to find y . Here I use an integrating factor. Since $y' + e^x y = e^x$ is already in the proper form, we set

$$I(x) = e^{\int e^x dx} = e^{e^x}.$$

Multiplying both sides by $I(x)$ and integrating, we get

$$e^{e^x} y = \int e^{e^x} e^x dx.$$

Making the u -substitution $u = e^x$, we have $du = e^x dx$ and hence

$$e^{e^x} y = \int e^u du = e^u = e^{e^x} + C,$$

or

$$y = 1 + C e^{-e^x}.$$