Quiz 3 Name: SOLUTIONS

Math 114 - Calculus II

October 1, 2008

Note: *In order to receive full credit, you must show work that justifies your answer.*

The following formula might be useful: $\text{proj}_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\mathbf{a}$

1. Let $A(1,-1,1)$ and $B(3,5,-2)$ be points in $\mathbb{R}^3$. Find a vector $\mathbf{v}$ in the same direction as $\overrightarrow{AB}$ that has length 3.

   $\overrightarrow{AB} = \langle 2, 6, -3\rangle$, so the length of $\overrightarrow{AB}$ is $\left|\overrightarrow{AB}\right| = \sqrt{4+36+9} = 7$.

   To create a vector in the direction of $\overrightarrow{AB}$ that has length 3, we scale $\overrightarrow{AB}$ by $\frac{3}{7}$, to obtain

$$\mathbf{v} = \frac{3}{7}\overrightarrow{AB} = \left\langle \frac{6}{7}, \frac{18}{7}, \frac{-9}{7} \right\rangle.$$

2. Suppose $\mathbf{a} = \langle 4, -2, -4\rangle$, $\mathbf{b} = \langle 2, y, -3\rangle$, and $\text{comp}_{\mathbf{a}}\mathbf{b}$ has length 2. Find y.

   Recall that $\text{comp}_{\mathbf{a}}\mathbf{b}$ is the length of the projection of $\mathbf{b}$ in the direction of $\mathbf{a}$.

   Thus, we want to solve:

$$\text{comp}_{\mathbf{a}}\mathbf{b} = \frac{|\mathbf{a}\cdot\mathbf{b}|}{|\mathbf{a}|} = \frac{|8-2y+12|}{\sqrt{16+4+16}} = \frac{|20-2y|}{6} = 2$$

   which implies $y = 4$.

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The following formula might be useful: $\text{proj}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\mathbf{a}$

1. Let $P(2,-1,3)$ and $Q(8,11,-2)$ be points in $\mathbb{R}^3$. Find the point that lies $\frac{2}{3}$ of the way from P to Q.

   One way to solve this problem is to note that the desired point is given by $\frac{2}{3}\overrightarrow{PQ} + \langle 2,-1,3\rangle$.

   Now $\overrightarrow{PQ} = \langle 6,12,-5\rangle$, so the point is

$$\frac{2}{3}\overrightarrow{PQ} + \langle 2,-1,3\rangle = \left\langle 4, 8, \frac{-10}{3}\right\rangle + \langle 2,-1,3\rangle = \left\langle 6, 7, \frac{-1}{3}\right\rangle.$$

2. If $\mathbf{a} = \langle -4,4,2\rangle$ and $\mathbf{b} = \langle 2,8,3\rangle$, find $\text{comp}_{\mathbf{a}}\mathbf{b}$.

   Recall that $\text{comp}_{\mathbf{a}}\mathbf{b}$ is the length of the projection of $\mathbf{b}$ in the direction of $\mathbf{a}$, or

$$\text{comp}_{\mathbf{a}}\mathbf{b} = \frac{|\mathbf{a}\cdot\mathbf{b}|}{|\mathbf{a}|}.$$

   Thus,

$$\text{comp}_{\mathbf{a}}\mathbf{b} = \frac{|-8+32+6|}{\sqrt{16+16+4}} = \frac{30}{6} = 5.$$