Quiz 5 Name: SOLUTIONS

Math 114 - Calculus II

October 15, 2008

Note: *In order to receive full credit, you must show work that justifies your answer.*

1. Find the limit, or say why it does not exist.

$$\lim_{t\to 0}\left\langle \ln(t+1), \frac{\sin t}{t}, \sqrt[3]{t}\right\rangle$$

Apply the limit to each component of the vector function:

$$\begin{aligned}\lim_{t\to 0}\left\langle \ln(t+1), \frac{\sin t}{t}, \sqrt[3]{t}\right\rangle &= \left\langle \lim_{t\to 0}\ln(t+1), \lim_{t\to 0}\frac{\sin t}{t}, \lim_{t\to 0}\sqrt[3]{t}\right\rangle \\ &= \langle 0, 1, 0\rangle\end{aligned}$$

2. At what point(s) does the helix $\mathbf{r}(t) = \langle 2\sin t, 2\cos t, t\rangle$ intersect the ellipsoid $x^2 + y^2 + 3z^2 = 16$?

First we find t such that $(2\sin t)^2 + (2\cos t)^2 + 3t^2 = 16$.

Since $\sin^2 t + \cos^2 t = 1$, we find that $t^2 = 4$, or $t = \pm 2$.

$t = 2$ corresponds to the point $(2\sin 2, 2\cos 2, 2)$.

$t = -2$ corresponds to the point $(2\sin(-2), 2\cos(-2), -2) = (-2\sin 2, 2\cos 2, -2)$.

Quiz 5 Name: SOLUTIONS

Math 114 - Calculus II

October 17, 2008

Note: *In order to receive full credit, you must show work that justifies your answer.*

1. Find the limit, or say why it does not exist.

$$\lim_{t\to\infty}\left\langle e^{-2t}, \frac{2t}{t+2}, \sin(2t)\right\rangle$$

Apply the limit to each component of the vector function:

$$\lim_{t\to\infty}\left\langle e^{-2t}, \frac{2t}{t+2}, \sin(2t)\right\rangle = \left\langle \lim_{t\to\infty} e^{-2t}, \lim_{t\to\infty}\frac{2t}{t+2}, \lim_{t\to\infty}\sin(2t)\right\rangle$$

However, this limit does not exist because $\lim_{t\to\infty}\sin(2t)$ does not exist.

2. A particle follws a trajectory given by $\mathbf{r}(t) = \langle \cos t, \sin t, \sqrt{t}\rangle$ for $t \geq 0$. At what time t does the particle intersect the hyperboloid $2x^2 + 2y^2 - z^2 = 1$?

Substituting $x = \cos t$, $y = \sin t$, and $z = \sqrt{t}$ into the equation of the hyperboloid, we find

$$2\cos^2 t + 2\sin^2 t - t = 1.$$

Since $\sin^2 t + \cos^2 t = 1$, we solve for t and obtain $t = 1$.

So the particle intersects the hyperboloid at time 1 and at the point $(\cos 1, \sin 1, 1)$.