Quiz 7

Name: SOLUTIONS

Math 114 - Calculus II
October 29, 2008

Note: *In order to receive full credit, you must show work that justifies your answer.*

Possibly useful formulas:

$$L = \int_a^b |\mathbf{r}'(t)| dt \qquad \mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} \qquad \mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} \qquad \kappa = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

1. Find the arc length of the curve $\mathbf{r}(t) = \langle 2t, \frac{4}{3}t^{3/2}, \frac{1}{2}t^2 \rangle$ from $t = 0$ to $t = 2$.

   Differentiate: $\mathbf{r}'(t) = \langle 2, 2t^{1/2}, t \rangle$.

   Arc length is given by

   $$L = \int_0^2 |\mathbf{r}'(t)| dt = \int_0^2 \sqrt{4 + 4t + t^2}\ dt = \int_0^2 (2 + t)\ dt = \left[2t + \frac{1}{2}t^2 \right]_0^2 = 6$$

2. Find the equation of the normal plane to the curve $\mathbf{r}(t) = \langle 2\sin t, 2\cos t, 4\sin^2 t \rangle$ at the point $(1, \sqrt{3}, 1)$.

   The normal vector to the normal plane is the tangent vector to the curve.

   The tangent vector to $\mathbf{r}(t)$ is given by $\mathbf{r}'(t) = \langle 2\cos t, -2\sin t, 8\sin t \cos t \rangle$.

   We need to find the value of t that gives $\mathbf{r}(t) = (1, \sqrt{3}, 1)$. Solving $2\sin t = 1$ and $2\cos t = \sqrt{3}$, we find $t = \frac{\pi}{6}$.

   The tangent vector to the curve at $(1, \sqrt{3}, 1)$ is $\mathbf{r}'\left(\frac{\pi}{6}\right) = \langle \sqrt{3}, -1, 2\sqrt{3} \rangle$.

   Thus, the normal plane has equation

   $$\sqrt{3}(x - 1) - 1(y - \sqrt{3}) + 2\sqrt{3}(z - 1) = 0$$

   or equivalently

   $$\sqrt{3}x - y + 2\sqrt{3}z = 2\sqrt{3}.$$

Quiz 7 Name: SOLUTIONS

Math 114 - Calculus II

October 31, 2008

Note: *In order to receive full credit, you must show work that justifies your answer.*

Possibly useful formulas:

$$L = \int_a^b |\mathbf{r}'(t)| dt \qquad \mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} \qquad \mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} \qquad \kappa = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

1. Find the arc length of the curve $\mathbf{r}(t) = \langle 3t, 2\cos t, 2\sin t\rangle$ from $t = 0$ to $t = \frac{\pi}{2}$.

   Differentiate: $\mathbf{r}'(t) = \langle 3, -2\sin t, 2\cos t\rangle$.

   Arc length is given by

$$L = \int_0^{\pi/2} |\mathbf{r}'(t)| dt = \int_0^{\pi/2} \sqrt{9 + 4\sin^2 t + 4\cos^2 t}\ dt = \int_0^{\pi/2} \sqrt{13}\ dt = \frac{\pi\sqrt{13}}{2}$$

2. Find all points on the graph of $y = \cos x$ such that the curvature is zero.

   First we need to parametrize the curve. Since y depends on x, let x be the parameter t, and the parametrization is

$$\mathbf{r}(t) = \langle t, \cos t, 0\rangle.$$

   Differentiate: $\mathbf{r}'(t) = \langle 1, -\sin t, 0\rangle \qquad \mathbf{r}''(t) = \langle 0, -\cos t, 0\rangle$.

   Then the curvature is given by

$$\kappa = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3} = \frac{|\langle 1, -\sin t, 0\rangle \times \langle 0, -\cos t, 0\rangle|}{|\langle 1, -\sin t, 0\rangle|^3} = \frac{|\langle 0, 0, -\cos t\rangle|}{(1 + \sin^2 t)^{3/2}} = \frac{|\cos t|}{(1 + \sin^2 t)^{3/2}}.$$

   Curvature is zero only when the numerator above is zero, that is, when $\cos t = 0$. This happens when $t = \frac{\pi}{2} + n\pi$ for any integer n.