Quiz 8

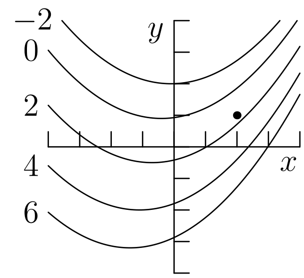
Name: SOLUTIONS

Math 114 - Calculus II
November 5, 2008

Note: *In order to receive full credit, you must show work that justifies your answer.*

Possibly useful formula: $z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$

1. The graph below is the contour map of a surface $z = f(x, y)$. The dot indicates the point $(2, 1)$.



   (a) What is the sign of $f_x(2, 1)$? How do you know?

   $f_x(2, 1)$ is **positive** because the surface is increasing along the cross section $y = 1$ at $x = 2$. (The contour to the left has height 0, while the contour to the right has height 2.)

   (b) What is the sign of $f_{xx}(2, 1)$? How do you know?

   $f_{xx}(2, 1)$ is **positive** because the surface is concave up along the cross section $y = 1$ at $x = 2$. (The contours get closer together as x increases, which indicates the slope is increasing.)

2. A surface is given by $z = g(x, y)$. Suppose $g(3, 5) = 2$ and $\mathbf{n} = \langle 1, 2, 1 \rangle$ is the vector normal to g at the point $(3, 5, 2)$. (The normal vector to a surface is the normal vector to the tangent plane of the surface.) Use this information to estimate $g(3.1, 5.2)$.

   The tangent plane to the surface at $(3, 5, 2)$ is $1(x - 3) + 2(y - 5) + 1(z - 2) = 0$, or $z = 15 - x - 2y$. This tangent plane approximates the surface for points near $(3, 5, 2)$.

   We can approximate $g(3.1, 5.2)$ by finding the value of z on the plane when $x = 3.1$ and $y = 5.2$:

$$g(3.1, 5.2) \approx 15 - 3.1 - 2(5.2) = 1.5.$$

Quiz 8

Name: SOLUTIONS

Math 114 - Calculus II
November 7, 2008

Note: *In order to receive full credit, you must show work that justifies your answer.*

Possibly useful formula: $z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$

1. The following table displays values of $f(x, y)$ for various values of x and y:

	$y=0$	$y=1$	$y=2$	$y=3$
$x=5$	4	1	-2	-3
$x=10$	6	4	2	1
$x=15$	9	5	4	2

Estimate the values of $f_x(10, 2)$ and $f_y(5, 1)$.

To estimate $f_x(10, 2)$, we can look at the column where $y = 2$ and average the slopes on either side of $x = 10$:

$$f_x(10,2) \approx \frac{1}{2}\left(\frac{f(10,2) - f(5,2)}{10-5} + \frac{f(15,2) - f(10,2)}{15-10}\right) = \frac{1}{2}\left(\frac{2+2}{5} + \frac{4-2}{5}\right) = \frac{3}{5}$$

To estimate $f_y(5, 1)$, we can look at the row where $x = 5$ and average the slopes on either side of $y = 1$:

$$f_y(5,1) \approx \frac{1}{2}\left(\frac{f(5,1) - f(5,0)}{1-0} + \frac{f(5,2) - f(5,1)}{2-1}\right) = \frac{1}{2}\left(\frac{1-4}{1} + \frac{-2-1}{1}\right) = -3$$

2. Suppose $\mathbf{v} = \langle 4, 0, 3\rangle$ and $\mathbf{w} = \langle 0, 5, 2\rangle$ are tangent to a surface at the point $P = (4, 2, 1)$. Find the equation of the tangent plane to the surface at P.

The normal vector to the surface at P must be orthogonal to both $\mathbf{v}$ and $\mathbf{w}$. If $\mathbf{n}$ the normal vector, then $\mathbf{n} = \mathbf{v} \times \mathbf{w} = \langle -15, -8, 20\rangle$.

Then the equation of the tangent plane is

$$-15(x-4) - 8(y-2) + 20(z-1) = 0 \quad \text{or} \quad -15x - 8y + 20z = -56.$$

Alternately, since $\mathbf{v}$ has zero y-component, we can conclude that $f_x(4, 2, 1) = \frac{3}{4}$. Similarly, $\mathbf{w}$ has zero x-component, so $f_y(4, 2, 1) = \frac{2}{5}$. We can then write the equation of the tangent plane using the formula at the top of the page:

$$z - 1 = \frac{3}{4}(x-4) + \frac{2}{5}(y-2).$$