Quiz 1 Name: _____

Math 240 - Calculus III

January 27, 2009

Note: *In order to receive full credit, you must show work that justifies your answer.*

1. Solve the following system:

$$\begin{aligned} a + b + c &= 5 \\ a + 2b + 3c &= 12 \\ 2a - b - c &= 1 \end{aligned}$$

Solution: We form the augmented matrix and row-reduce

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 1 & 2 & 3 & 12 \\ 2 & -1 & -1 & 1 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & 2 & 7 \\ 0 & -3 & -3 & -9 \end{array}\right] \to$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & 3 & 12 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

Thus, the solution is $a = 2$, $b = -1$, and $c = 4$. Substituting these values back into the original equations, we see that our answer is correct.

2. Find the rank of the matrix $\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 3 \end{bmatrix}$.

Solution: The row-reduced form of $\mathbf{A}$ has two nonzero rows:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus, the rank of $\mathbf{A}$ is 2.

Quiz 1 Name: _____

Math 240 - Calculus III

January 29, 2009

Note: *In order to receive full credit, you must show work that justifies your answer.*

1. Find the rank of the matrix $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{bmatrix}$.

   Solution: The first two rows of $\mathbf{A}$ are linearly independent (why?), so the rank of $\mathbf{A}$ is at least 2. However, $\mathbf{A}$ has only two columns, so its rank is at most 2. Therefore, the rank of $\mathbf{A}$ is 2.

   Alternately, we could row-reduce $\mathbf{A}$ and see that the reduced form has 2 nonzero rows.

2. Find a number n such that the vectors $\langle 1, 2, 0\rangle$, $\langle 0, 2, 3\rangle$, and $\langle 3, 2, n\rangle$ are linearly *dependent*.

   Solution: Consider the matrix whose rows are the given vectors: $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 3 \\ 3 & 2 & n \end{bmatrix}$.

   The row vectors are linearly dependent if and only if we can row reduce and obtain at least one row of zeros. Row-reducing:

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 3 \\ 3 & 2 & n \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 3 \\ 0 & -4 & n \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & n+6 \end{bmatrix}$$

   The reduced form of the matrix has a row of zeros if and only if $n = -6$.

   Therefore, the given vectors are linearly dependent when $n = -6$.