Quiz 3 Name: _____

Math 240 - Calculus III

February 17, 2009

Note: *In order to receive full credit, you must show work that justifies your answer.*

1. Find the general solution of the following second-order differential equation:

$$y'' - 9y = 0$$

Solution: The auxiliary equation is $m^2 - 9m = 0$, which has solutions $m = -3, 3$.

This implies that $y_1(x) = e^{-3x}$ and $y_2(x) = e^{3x}$ are both solutions to the given differential equation.

Therefore, the general solution is

$$y(x) = c_1 e^{-3x} + c_2 e^{3x}.$$

2. Find the general solution of the given system:

$$\frac{dx}{dt} = 2x + y, \quad \text{and} \quad \frac{dy}{dt} = 4y.$$

Solution: This system corresponds to the matrix equation

$$\mathbf{X}' = \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix} \mathbf{X}.$$

Since the matrix is diagonal, its eigenvalues are $\lambda_1 = 2$ and $\lambda_2 = 4$.

An eigenvector for $\lambda_1 = 2$ is $\mathbf{v}_1 = \langle 1, 0 \rangle$, and an eigenvector for $\lambda_2 = 4$ is $\mathbf{v}_2 = \langle 1, 2 \rangle$.

Therefore, the general solution is

$$\mathbf{X} = c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{4t}.$$

Quiz 3 Name: _____

Math 240 - Calculus III

February 19, 2009

Note: *In order to receive full credit, you must show work that justifies your answer.*

1. Determine whether the following functions are linearly dependent or linearly independent on the interval $(-\infty, \infty)$:

$$f_1(x) = 5, \quad f_2(x) = \cos^2(x), \quad f_3(x) = \sin^2(x)$$

Solution: The given functions are linearly dependent, as shown by the following linear combination:

$$f_1(x) - 5f_2(x) - 5f_3(x) = 5 - 5\cos^2(x) - 5\sin^2(x) = 0.$$

2. Solve the following initial value problem:

$$y'' - 4y' - 5y = 0, \quad y(0) = 2, \quad y'(0) = -8$$

Solution: The auxiliary equation is $m^2 - 4m - 5 = 0$, which has solutions $m = -1, 5$. This implies that $y_1(x) = e^{-x}$ and $y_2(x) = e^{5x}$ are both solutions to the given differential equation, and the general solution is

$$y(x) = c_1 e^{-x} + c_2 e^{5x}.$$

The initial conditions give the system

$$\begin{aligned} y(0) &= c_1 + c_2 = 2 \\ y'(0) &= -c_1 + 5c_2 = -8 \end{aligned}$$

Solving this system, we obtain $c_1 = 3$ and $c_2 = -1$, so the initial value problem has solution

$$y(x) = 3e^{-x} - e^{5x}.$$