Quiz 7

Name: ______________________________

Math 240 - Calculus III
March 31, 2009

Note: *In order to receive full credit, you must show work that justifies your answer.*

1. (a) (4 points) Define a function $f : \mathbb{R}^3 \to \mathbb{R}^3$ as the cross product of the input vector with the vector $\langle 1, 1, 0\rangle$. That is,

$$f(x, y, z) = \langle x, y, z\rangle \times \langle 1, 1, 0\rangle = \langle -z, z, x - y\rangle.$$

Compute the Jacobian $[Df] = \left[\frac{df_i}{dx_j}\right]_{i,j}$.

Solution:

$$[Df] = \left[\frac{df_i}{dx_j}\right]_{i,j} = \begin{bmatrix} \frac{df_1}{dx_1} & \frac{df_1}{dx_2} & \frac{df_1}{dx_3} \\ \frac{df_2}{dx_1} & \frac{df_2}{dx_2} & \frac{df_2}{dx_3} \\ \frac{df_3}{dx_1} & \frac{df_3}{dx_2} & \frac{df_3}{dx_3} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

Note that the Jacobian $[Df]$ does not depend on the inputs $\langle x, y, z\rangle$.

(b) (4 points) Suppose the inputs are $\langle x, y, z\rangle = \langle 0, 1, 1\rangle$ and the rates of change of the inputs are $\langle \dot{x}, \dot{y}, \dot{z}\rangle = \langle 1, 0, 2\rangle$. Find the rates of change of the outputs.

Solution:

$$\left[Df\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}\right]\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$$

2. (2 points) Write at least one thing you learned from the midterm last week.

Solution: Exams can be learning experiences just like homework and quizzes! What did you learn?

Quiz 7 Name: ______________________

Math 240 - Calculus III

April 2, 2009

Note: *In order to receive full credit, you must show work that justifies your answer.*

1. (a) (4 points) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined

$$f\begin{pmatrix} r \\ \theta \end{pmatrix} = \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix}$$

Compute the Jacobian $[Df] = \left[\frac{df_i}{dx_j}\right]_{i,j}$.

Solution:

$$[Df] = \left[\frac{df_i}{dx_j}\right]_{i,j} = \begin{bmatrix} \frac{df_1}{dx_1} & \frac{df_1}{dx_2} \\ \frac{df_2}{dx_1} & \frac{df_2}{dx_2} \end{bmatrix} = \begin{bmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{bmatrix}$$

(b) (4 points) Suppose the inputs are $\langle r, \theta\rangle = \langle 1, \frac{\pi}{2}\rangle$ and the rates of change of the inputs are $\langle \dot{r}, \dot{\theta}\rangle = \langle 0, 4\pi\rangle$. Find the rates of change of the outputs.

Solution:

$$\left[Df\begin{pmatrix} 1 \\ \frac{\pi}{2} \end{pmatrix}\right]\begin{pmatrix} \dot{r} \\ \dot{\theta} \end{pmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{pmatrix} 0 \\ 4\pi \end{pmatrix} = \begin{bmatrix} -4\pi \\ 0 \end{bmatrix}$$

Let $x = r\cos\theta$ and $y = r\sin\theta$. Then $\dot{x} = -4\pi$ and $\dot{y} = 0$.

2. (2 points) Write at least one thing you learned from the midterm last week.

Solution: Exams can be learning experiences just like homework and quizzes! What did you learn?