Quiz 9 Name: ____________________________

Math 240 - Calculus III

April 14, 2009

Note: *In order to receive full credit, you must show work that justifies your answer.*

1. (6 points) Evaluate the integral

$$\int_C (2x-y)dx + (x+3y)dy$$

where C is the path given by $y = 1 - x^2$ from $(0,1)$ to $(1,0)$.

Solution: The path is parametrized by $x = t$, $y = 1 - t^2$, for $0 \leq t \leq 1$. Thus,

$$\begin{aligned}\int_C (2x-y)dx + (x+3y)dy &= \int_0^1 \left(2t - (1-t^2)\right) dt + \left(t + 3(1-t^2)\right)(-2t)dt \\ &= \int_0^1 \left(6t^3 - t^2 - 4t - 1\right) dt \\ &= -\frac{11}{6}\end{aligned}$$

2. (4 points) For what value A is the integral

$$\int_C (2x-y)dx + (Ax+3y)dy$$

independent of path? What is the corresponding potential function?

Solution: Let $P = 2x - y$ and $Q = Ax + 3y$. The integral is independent of path if $P_y = Q_x$, which occurs when $A = -1$.

The potential function is a function $\phi(x,y)$ such that $\phi_x = P$ and $\phi_y = Q$. Examining antiderivatives, we find

$$\phi(x,y) = x^2 - xy + \frac{3}{2}y^2 + C$$

for any constant C.

Quiz 9 Name: ____________________

Math 240 - Calculus III

April 16, 2009

Note: *In order to receive full credit, you must show work that justifies your answer.*

1. (7 points) Find the work done by the force $\mathbf{F}(x,y) = -x^2\mathbf{i} + xy\mathbf{j}$ acting along the curve given by $x = \cos t$, $y = \sin t$ from $(1,0)$ to $(0,1)$.

   Hint: Work $= \int_C \mathbf{F} \cdot d\mathbf{r}$

   Solution: The path is $\mathbf{r}(t) = \langle \cos t, \sin t \rangle$ for $0 \leq t \leq \frac{\pi}{2}$.

$$\begin{aligned}
\int_C \mathbf{F} \cdot d\mathbf{r} &= \int_0^{\pi/2} \langle -x^2, xy \rangle \cdot \langle -\sin t, \cos t \rangle dt \\
&= \int_0^{\pi/2} \langle -\cos^2 t, \cos t \sin t \rangle \cdot \langle -\sin t, \cos t \rangle dt \\
&= \int_0^{\pi/2} \left(\sin t \cos^2 t + \cos^2 t \sin t \right) dt \\
&= 2 \int_0^{\pi/2} \sin t \cos^2 t \; dt \\
&= \frac{2}{3}.
\end{aligned}$$

2. (3 points) Is the integral you evaluated above independent of path? How do you know?

   Solution: No, the integral is not independent of path because the vector field given by $\mathbf{F}(x,y)$ is not the gradient of any function. We know this because $P_y \neq Q_x$, where $\mathbf{F}(x,y) = \langle P, Q \rangle = \langle -x^2, xy \rangle$.

   Another way of saying this is that $-x^2\, dx + xy\, dy$ is not an exact differential.