(Math 170) Homework 11:

Due April 13, 2007

Exercise 1: Choose some partial (but not total) recursive function of 1 variable, call it $f(n)$. Write a state diagram for a well-formed Turing machine which represents the function.

In other words, given a tape with a single block of 1's representing the number n with the head of your Turing machine at the left most 1, your Turing machine should halt if and only if $f(n)$ is defined. Further, if your Turing machine halts on a tape representing n, then the tape when it halts should have a single block of 1's representing the number $f(n)$ and the head should be at the left most 1 in the block.

Exercise 2: Write a state diagram for aa 2-Tape Turing machine T such that if

- On the main tape there is a single block of n 1's with the head at the left most 1.
- On the second tape there is a single block of m 1's with the head at the left most 1.

Then the Turing machine will halt with a single block of $m+n$ 1's on the main tape and the head of the main tape at the left most 1 (note we don't care what the end condition of the second tape is).

Let A be a set of natural numbers. Recall that we say an infinite tape encodes a A if

- There is a single cell with the symbol $\Box$
- All cells to the left of the cell with $\Box$ are 0.
- $n \in A$ if and only if the $n+1$'s cell to the left of the cell with $\Box$ is a 1.
- $n \notin A$ if and only if the $n+1$'s cell to the left of the cell with $\Box$ is a 0.

Exercise 3: Let $A = \{0, 1, 2, 4, 8\}$. Find a tape which encodes A.

Exercise 4: Let $B = \{3, 4, 5, 6\}$. Find a tape which encodes B.

<u>Exercise 5:</u> Let $C = \{2n : n \text{ is a natural number}\}$, $D = \{n^2 : n \text{ is a natural number}\}$. Find a bijection from C to D.

<u>Exercise 6:</u> Find an injection from $\boldsymbol{N} = \{n : n \text{ is a natural number}\}$ to $[0, 1] = \{x : 0 \leq x \leq 1, x \text{ is a real number}\}$