(Math 170) Homework 4:

Due February 16, 2007

Exercise 1: Find the following up to 6 decimal places

(a) $[1], [1,4], [1,4,1]$ and $[1,4,1,4]$.

(b) $[1], [1,3], [1,3,1]$ and $[1,3,1,3]$.

(c) Which is larger?

Exercise 2: Find an expression of the form a/b where a, b are real numbers for

(a) $[1,4,1,4,\dots]$

(b) $[1,3,1,3,\dots]$

Let

$$q_n = \sum_{k=1}^{n} \frac{1}{n!}$$

Exercise 3: Prove that $\{q_n\}_{n\in\mathbb{N}}$ is a Cauchy Sequence.

Exercise 4: Let $r = \lim_{n\to\infty} q_n$. Approximate r to 6 decimal places.

Let $\{q_n\}_{n\in\mathbb{N}}, \{p_n\}_{n\in\mathbb{N}}$ be Cauchy Sequences which are equivalent. Further let $\{a_n\}_{n\in\mathbb{N}}, \{b_n\}_{n\in\mathbb{N}}$ also be Cauchy Sequences which are equivalent.

Exercise 5: (a) Show, using properties of sums of Cauchy Sequences, that $\{q_n - a_n\}_{n\in\mathbb{N}}$ is equivalent to $\{p_n - b_n\}_{n\in\mathbb{N}}$.

(a) Show, without using properties of sums of Cauchy sequences discussed in class, that $\{q_n - a_n\}_{n\in\mathbb{N}}$ is equivalent to $\{p_n - b_n\}_{n\in\mathbb{N}}$.

Exercise 6: (2 Bonus Points) Show $\{q_n \cdot a_n\}_{n\in\mathbb{N}}$ is equivalent to $\{p_n \cdot b_n\}_{n\in\mathbb{N}}$