(Math 170) Homework 8:

Due March 23, 2007

Exercise 1: Find the greatest common divisor of 210 and 1573.

Exercise 2: If $d = gcd(210, 1573)$, use Euclid's Algorithm to find $a, b \in \mathbb{Z}$ such that $210a + 1573b = d$.

Exercise 3:
(a) What is $\phi(97)$?
(b) What is $\phi(143)$?
(c) What is $\phi(504)$?

Exercise 4: What is the function encoded by

$$2^3 \cdot 3^2 \cdot 5^1 \cdot 7^2 \cdot 11^{225}$$

Exercise 5: Find a number n which encodes the function $Pred(x)$ (i.e. $F(n) = Pred(x)$) where $Pred(x)$ is defined by primitive recursion as the function such that

- $Pred(0) = 0$
- $Pred(x + 1) = \Pi_2(Pred(x), x)$

Note here Π_2 is the function which takes two arguments and projects on to the second one.

It may be useful to recall our method of encoding primitive recursive functions by numbers. Let n be a natural number and let $F(n)$ be the function n represents. Then, if $n = 2^{k_0} \cdot 3^{k_1} \cdot 5^{k_2} \cdots$ is a prime factorization of n we determine the function $F(n)$ as follows.

- If $k_0 = 0$ then $F(n)$ is a Constant Function
  - $k_1 =$ number of variables
  - $k_2 =$ constant.

  So $F(n)$ is the function $f(x_1, \ldots, x_{k_1}) = k_2$
- If $k_0 = 1$ then $F(n)$ is a Successor Function. So $F(n)$ is the function $f(x) = x + 1$
- If $k_0 = 2$ then $F(n)$ is a Projection Function
  - $k_1 =$ number of variables
  - $k_2 =$ variable projected onto.

  So $F(n)$ is the function $\pi_{k_2}(x_1, \ldots, x_{k_1}) = x_{k_2}$
- If $k_0 = 3$ then $F(n)$ is a Composition.
  - $k_1 =$ number of variables of inner functions.
  - $k_2 =$ number of variables of outer function.
  - $F(k_3) =$ outer function f.
  - $F(k_{3+i}) =$ ith inner function h_i

  So $F(n)$ is the function $f(h_1(x_1, \ldots, x_{k_1}), \ldots, h_{k_2}(x_1, \ldots, x_{k_1}))$
- If $k_0 = 4$ then $F(n)$ is obtained by Primitive Recursion.
  - $k_1 =$ number of variables of the base case function $f(x_1, \ldots, x_{k_1})$.
  - $F(k_2) =$ base case function, $f(x_1, \ldots, x_{k_1})$.
  - $F(k_3) =$ function used for iteration, $g(z, n, x_1, \ldots, x_{k_1})$

  So $F(n)$ is the function $h(n, (x_1, \ldots, x_{k_1})$ define so that
  - $h(0, x_1, \ldots, x_{k_1}) = f(x_1, \ldots, x_{k_1})$

  - $h(n+1, (x_1, \ldots, x_{k_1}) = g(h(n, x_1, \ldots, x_{k_1}), n, x_1, \ldots, x_{k_1})$

- If at any point the number of variables is inconsistent with the function, then $F(n)$ is the constant function with zero variables which has value 0