(Math 170) Homework 9:

Due March 30, 2007

Definition 0.0.1. Let $G(m,n)$ be the function defined as follows. If $F(m)$ is the function $f(x_1,\ldots,x_l)$ and $n = 2^{k_0} \cdot 3^{k_1} \cdots Prime(r)^{k_r}$ then

- $G(m,n) = 0$ if $f(k_0,\ldots,k_l)$ is undefined
- $G(m,n) = 1$ if $f(k_0,\ldots,k_l)$ is defined

Definition 0.0.2. Let $\mathcal{H} = \{$Functions obtained from the recursive functions and G by the recursive operations (i.e. Composition, Primitive Recursion, and the μ operator).$\}$

Exercise 1: Describe a method for coding a function in $\mathcal{H}$ by a natural number. I.e. Find a function H from the natural numbers to $\mathcal{H}$ such that for every function $h \in \mathcal{H}$ there is at least one natural number n such that $H(n) = h$.

Your method should agree with the method described in class on all natural numbers such that 2^6 does not divide n.

Exercise 2: Let $J(m,n)$ be the function defined as follows. If $H(m)$ is the function $h(x_1,\ldots,x_l)$ and $n = 2^{k_0} \cdot 3^{k_1} \cdots Prime(r)^{k_r}$ then

- $J(m,n) = 0$ if $f(k_0,\ldots,k_l)$ is undefined
- $J(m,n) = 1$ if $f(k_0,\ldots,k_l)$ is defined

(Note here H is the function defined in the previous exercise).
Is $J \in \mathcal{H}$? Prove your answer. (Hint: Use a diagonal argument)

Exercise 3: Give an example of a partial (or total) function which is recursive but not primitive recursive. Justify your answer.

Exercise 4: Let $m(x,y) = x \cdot y$. Find a definition of m in terms of the basic primitive recursive operations and functions.

Exercise 5: Which of the following sets are recursive? Which are recursively enumerable? Which are neither? You may assume the Church-Turing Thesis.

(a) $\{2^a \cdot 3^b \cdot 5^c \cdot 7^d : a = b! + c \cdot d\}$

(b) $\{2^n \cdot 3^m$: where n is the mth digit of $\pi\}$.

(c) $\{2^n \cdot 3^m$: where $F(n)$ and $F(m)$ are different total recursive functions$\}$