(Math 170) Homework 1:

Due September 16, 2007

Exercise 1: **Out of sight but not out of mind.** The infamous band Slippery Even When Dry ended their concert and checked into the Fuzzy Fig Motel. The guys in the band (Spike, Slip, and Milly) decided to share a room. They were told by Chip, the night clerk who was taking a home study course on animal husbandry, that the room cost 25$ for the night.

Milly, who took care of the finances, collected 10$ from each band member and gave chip 30$. Chip handed Milly the change, 5$ in singles. Milly, knowing how bad slip and Spike were at arithmetic, pocketed two of the dollars, turned to the others and said, "Well guys, we got 3$ change, so we each get a buck back". He then gave each of the other two members a dollar and pocketed the last one for himself.

Once the band members left the office, Chip, who witnessed this little piece of deception, suddenly realized that something strange had just happened. Each of the three band members first put in 10$ so there was a total of 30$ at the start. Then Milly gave each guy and himself 1$ back. That means each person put in only 9$, which is a total of 27$ (9$ from each of the three). But Milly had skimmed off 2$, so that gives a total of 29$. But there was 30$ to start with. Chip wondered what happened to that extra dollar and who had it. Can you please resolve and explain the issue to Chip?

Exercise 2: **The cannibals and the missionaries.** In 1853 in the wilds of central Iowa, three missionaries and three cannibals were walking in a group. The missionaries were trying to convert the cannibals to their religion, while the cannibals were looking for a chance to practice their culture on the missionaries. After a time, they all came to a river that they wished to cross. None of the six could swim, but all could row. Fortunately , on the river bank was a small rowboat available for use.

Since the boat was small and the cannibals and the missionaries were all on the large side, it was clear that only two persons could cross at one time. It was late in the day and neither cannibals nor missionaries had eaten much recently, and the missionaries began to notice that the cannibals were indicating greater and greater appreciation for the missionaries ample girths. The missionaries decided that being prudent was better than being a main course, so they agreed that at no time would they allow any group of missionaries to be outnumbered by cannibals during the crossing. For their part, the cannibals did not fear being outnumbered by the missionaries because they realized that an excess of missionaries would result only in more discussion among the missionaries, thus relieving the cannibals of the burden of polite conversation.

How do the cannibals and the missionaries all cross the river using only the one boat yet at no time letting the cannibals outnumber the missionaries on either side of the river?

Exercise 3: **Getting a pole on a bus.** For his 13th birthday, Adam was allowed to travel down to Sarah's Sporting Goods store to purchase a brand new fishing pole. With great excitement and anticipation, Adam boarded the bus on his own and arrived at Sarah's store. Although the collection of fishing poles was tremendous, there was only one pole for Adam and he bought it" a 5-foot, one-piece fiberglass "Trout Troller 570" fishing pole.

When Adam's return bus arrived, the driver reported that Adam could not board the bus with a fishing pole. Objects longer than 4 feet were not allowed on the bus. Adam remained at the bus stop holding his beautiful 5-foot Trout Troller. Sarah, who had observed the whole ordeal, rushed out and said, "We'll get your fishing pole on the bus!" Sure enough, when the same bus and the same driver returned, Adam boarded the bus with his fishing pole, and the driver welcomed him aboard with a smile. How was Sarah able to have Adam board the bus with his 5-foot fishing pole without breaking or bending the bus-line rules or the pole?

Exercise 4: **Twenty-nine is fine.** Find the most interesting property you can, unrelated to size, that the number 29 has and that 27 does not have.

Exercise 5: **Perfect numbers.** The only natural numbers that divide evenly into 6, other than 6 itself, are 1, 2, and 3. Notice that the sum of all those numbers equals the original number 6 ($1+2+3=6$). What is the next number that has the property of equaling the sum of all the natural numbers, other than itself, that divide evenly into it? Such numbers are called *perfect numbers*. No one knows whether or not there are infinitely many perfect numbers. In fact, no one knows whether there are *any* odd perfect numbers. These two unsolved mysteries are examples of long-standing open questions in the theory of numbers.

Exercise 6: **Sock hop.** You have 10 pairs of socks, five black and five blue, but they are not paired up. Instead they are all mixed up in a drawer. It's early in the morning, and you don't want to turn on the lights in your dark room. How many socks must you pull out to guarantee that you have a pair of one color? How many must you pull out to have two good pairs (each pair is the same color)? How many must you pull out to be certain you have a pair of black socks?

Exercise 7: **The last one.** Here is a game to be played with natural numbers. You start with any number. If the number is even, you divide it by 2. If the number is odd, you triple it (multiply it by 3), and then add 1. Now you repeat the process with this new number. Keep going. You win (and stop) if you get 1. For example, if we start with 17 we would have

$$17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1 \rightarrow \text{we see a 1, so we win!}$$

Play four rounds of this game starting with the numbers $19, 11, 22$, and 30. Do you think you will always win no matter what number you start with? No one knows the answer!

Exercise 8: Tic-tac-toe is a well known childrens game. However if we make a small modification and say that first player wins whenever it is a tie, then the game becomes a two player, perfect information, deterministic game without ties. So we know that one of the players has a winning strategy. Find out which player has a winning strategy and describe it.